Math Fundamentals Supplement

Both the Quant and Data Insights sections of the GMAT rely on math concepts, and *The GMAT Mentor* is packed with strategies and techniques to help you handle whatever the test throws your way. But if your fundamentals are rusty—or if math was never your thing—jumping straight into the program can feel like diving into the deep end before you're ready.

This supplement eases you into the shallow end first. Here, you can get comfortable with the "water," take your time, and rebuild conceptual understanding—without the pressure of keeping your head above the surface. That understanding will free up the mental bandwidth you'll need to get the most out of the strategy lessons in *The GMAT Mentor: Quant and Data Insights* (available at Amazon, if you don't already have it).

I won't cover all the math fundamentals here, only the essentials you need to start strong. Once you're able to "swim"—or at least doggy-paddle with confidence—*The GMAT Mentor* will guide you through the rest, blending new math concepts with GMAT-specific tactics so you can apply your learning to realistic problems right away. As your skills grow, so will your confidence—and you'll know where and how to use every concept the test expects from you.

Ready? Let's dive in.

Contents

Math Fundamentals Diagnostic Quiz

This quiz will tell you what kind of math fundamentals review you need (if any). Even if you're fairly comfortable with math concepts, I'd still recommend taking the quiz to see if you have any gaps that can be addressed by this supplement. This is the same quiz that appears in Chapter 2 of *The GMAT Mentor: Quant and Data Insights*.

Give yourself up to twenty minutes to take the diagnostic quiz. Don't use a calculator.

Numbers and Digits

1. In the set of numbers $\left\{-2, 0, \frac{3}{4}, \sqrt{2}, 7, 10.5\right\}$, how many are integers?
2. In the number 8,192.635, what is the value of the digit in the hundredths place?

Calculations

3. Calculate $13.7 - 35.6$
4. Calculate -3.2×-7
5. What is the quotient and remainder of $116 \div 9$?
6. Calculate $50.56 \div 0.8$

Arithmetic Shortcuts

7. Calculate 69×6 without doing any written work.
8. Find a rough estimate for 39.1×52.36 without doing any written work.

Fractions and Percents

9. Put $\frac{24}{27}$ in lowest terms.
10. What is 0.3% of 1400?

Expressions

11. What is 3^4?
12. Calculate $|3(2^2 - 5)|$
13. Calculate $\frac{3^2 - 1}{\sqrt{16}}$
14. Simplify $3x^2 + 2x^2 + 2x^2y$
15. Simplify $2x(y + 2z)$

Equations and Inequalities

16. If $2x - 2 = 4$, what is the value of $x^2 + 4x + 6$?
17. Isolate x in the following inequality: $-3x < 9$
18. If John is 3 years older than twice Mary's age, and John is 25 years old, how old is Mary?
19. If Ray has half as many books as Jose, and Jose has 3 times as many books as Gary, Ray has how many times the number of books as Gary?
20. What is the average of 9, 10, and 12?

Calculator Use:
You don't get a calculator on the Quant section, so get used to doing math without one. You do get a calculator on the Data Insights section, but even there the questions are usually—though not always—more efficient to solve by hand.

Math Fundamentals Diagnostic Quiz Solutions

Numbers and Digits

1. In the set of numbers $\left\{-2, 0, \frac{3}{4}, \sqrt{2}, 7, 10.5\right\}$, how many are integers?

Integers are whole numbers (including zero) and negative whole numbers. The integers are –2, 0, and 7 $\Longrightarrow$ 3 integers

2. In the number 8,192.635, what is the value of the digit in the hundredths place?

The hundredths place is two to the right of the decimal point. Here, its value is $3 \times 0.01 \Longrightarrow 0.03$ (or $\frac{3}{100}$)

Calculations

3. Calculate 13.7 – 35.6

Here, you're subtracting a larger positive number from a smaller positive number. The result will be negative. To calculate, flip the numbers, line up the decimals, and perform the subtraction, borrowing as needed:

$$\begin{array}{r} \overset{4\ 16}{35.6} \\ \underline{-13.7} \\ 21.9 \end{array}$$

Remember to make the result negative $\Longrightarrow$ –21.9

4. Calculate -3.2×-7

Since you're multiplying two negatives, the product will be positive. Set up the problem and multiply, carrying as needed:

$$\begin{array}{r} \overset{1}{-3.2} \\ \underline{\times -7} \\ 224 \end{array}$$

Since there is one decimal place, move the decimal point over one place to $\Longrightarrow$ 22.4

5. What is the quotient and remainder of 116 ÷ 9?

Use long division to solve:

$$\begin{array}{r} 12 \\ 9\overline{)116} \\ \underline{-9} \\ 26 \\ \underline{-18} \\ 8 \end{array}$$

The quotient is $\Longrightarrow$ 12 with a remainder of 8

6. Calculate 50.56 ÷ 0.8

First, move the decimal point one place to the right in both numbers so that you're dividing by an integer: 505.6 ÷ 8

To calculate, perform the long division:

$$\begin{array}{r} 63.2 \\ 8\overline{)505.6} \\ \underline{-48} \\ 25 \\ \underline{-24} \\ 16 \\ \underline{-16} \\ 0 \end{array}$$

The result is $\Longrightarrow$ 63.2

Arithmetic Shortcuts

7. Calculate 69×6 without doing any written work.

Since 69 is close to 70, calculate $70 \times 6 = 420$ and subtract one 6 → $420 - 6 \Longrightarrow 414$

8. Find a rough estimate for 39.1×52.36 without doing any written work.

Round 39.1 to 40 and round 52.36 to 50.

Then, $40 \times 50 \Longrightarrow 2{,}000$

Fractions and Percents

9. Put $\frac{24}{27}$ in lowest terms.

3 divides evenly into both numerator ($24 \div 3 = 8$) and denominator ($27 \div 3 = 9$), so you can rewrite the fraction as $\Longrightarrow \frac{8}{9}$

10. What is 0.3% of 1400?

0.3% is equivalent to 0.003. Then multiply, carrying as needed:

$$\begin{array}{r} \overset{1}{1400} \\ \underline{\times 0.003} \\ 4200 \end{array}$$

Then, move the decimal point over three places to give you 4.200 or simply $\Longrightarrow$ 4.2

Math Fundamentals Diagnostic Quiz Solutions (continued)

Expressions

11. What is 3^4?

This is equal to $3 \times 3 \times 3 \times 3 \implies 81$

12. Calculate $|3(2^2 - 5)|$

First, convert 2^2 to 4, giving you: $|3(4 - 5)|$

Then, calculate within the parentheses: $|3(-1)|$

Multiply 3×-1 to get: $|-3|$

The absolute value makes the result positive $\implies 3$

13. Calculate $\frac{3^2 - 1}{\sqrt{16}}$

In the numerator, convert 3^2 to 9, then subtract 1: $\frac{8}{\sqrt{16}}$

In the denominator, take the square root of 16: $\frac{8}{4}$

Divide 8 by 4 $\implies 2$

14. Simplify $3x^2 + 2x^2 + 2x^2y$

Since $3x^2$ and $2x^2$ are like terms, they can be added: $3x^2 + 2x^2 = 5x^2$. But $5x^2$ and $2x^2y$ are not like terms and can't be added $\implies 5x^2 + 2x^2y$

15. Simplify $2x(y + 2z)$

Distribute the $2x$ to both terms in the parentheses: $2x \cdot y + 2x \cdot 2z \implies 2xy + 4xz$

Equations and Inequalities

16. If $2x - 2 = 4$, what is the value of $x^2 + 4x + 6$?

First, isolate the variable: $2x - 2 = 4 \rightarrow 2x = 6 \rightarrow x = 3$

Then, substitute 3 for x: $(3)^2 + 4(3) + 6 \rightarrow 9 + 12 + 6 \implies 27$

17. Isolate x in the following inequality: $-3x < 9$

Divide both sides by -3 and flip the inequality sign: $\frac{-3x}{-3} > \frac{9}{-3}$ $\implies x > -3$

18. If John is 3 years older than twice Mary's age, and John is 25 years old, how old is Mary?

Translate this as: $j = 2m + 3$ and $j = 25$. Then, substitute 25 for j: $25 = 2m + 3$

Subtract 3 from both sides: $22 = 2m$. Divide both sides by 2, resulting in $\implies m = 11$

19. If Ray has half as many books as Jose, and Jose has 3 times as many books as Gary, Ray has how many times the number of books as Gary?

Translate this as $r = \frac{1}{2}j$ and $j = 3g$. Then substitute $3g$ for j in the first equation: $r = \frac{1}{2}(3g) \rightarrow r = \frac{3}{2}g$ or $r = 1.5g$ $\implies \frac{3}{2}$ or 1.5 times

20. What is the average of 9, 10, and 12?

Calculate $\frac{\textit{sum of terms}}{\textit{number of terms}}$ to get: $\frac{9+10+12}{3} \implies \frac{31}{3}$ or $10\frac{1}{3}$

How Did You Do?

The sections of the diagnostic (Numbers and Digits, Calculations, Arithmetic Shortcuts, Fractions and Percents, Expressions, and Equations and Inequalities) correspond to the remaining sections of the Math Fundamentals Supplement. Use your results on the quiz to guide your review of these sections. If you need to review the supplement, do so before starting Chapter 4 of the book:

Number Incorrect	Recommendation
0	If you got every question correct, nicely done! You probably don't need to review the supplement.
1–3	Review the sections in which you missed questions.
4–6	Review the whole supplement. You can skim through sections in which you missed no questions.
7 or more	Review every section of the supplement in detail.

Numbers and Digits

For the GMAT, you must be familiar with a number of mathematical definitions. The most basic definitions involve types of numbers you'll see on the test:

- *Positive numbers* are numbers greater than zero. *Negative numbers* are numbers less than zero. *Zero* is neither negative nor positive.

- *Whole numbers* are the basic counting numbers including zero (0, 1, 3, 50, 99, 274, etc.).

- *Integers* consist of whole numbers (as above) and *negative* whole numbers (–1, –3, –80, –121, etc.).

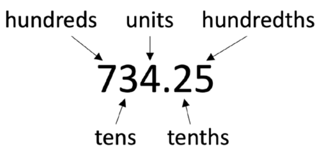
- *Real numbers* are both integers and the numbers that lie "in between" the integers, such as *decimals* (e.g., 1.875), *fractions* (e.g., $\frac{1}{2}$), and *irrational numbers* (e.g., $\sqrt{2}$ or π).

Digits

All integers and decimals are made up of *digits*, which are each a numeral from 0–9.

Example: The number 734.25 consists of five digits: 7, 3, 4, 2, and 5.

The digits have *places* that are named as follows:



Each of the digits represents a particular *value*:

- The digit 7, since it is in the hundreds place, has a value of 7×100, or 700

- The digit 3 has a value of $3 \times 10 = 30$

- 4 has a value of $4 \times 1 = 4$

- 2 has a value of $2 \times 0.1 = 0.2$

- 5 has a value of $5 \times 0.01 = 0.05$

All the digit values added together equal the value of the number:

$$\begin{array}{r} 700 \\ +30 \\ +4 \\ +0.2 \\ \underline{+0.05} \\ 734.25 \end{array}$$

If you added a digit in front of the 7, it would be the *thousands* digit. If you added a digit to the right of the 5, it would be the *thousandths* digit, and so forth.

Drill

Questions 1–5 refer to the following number: 4,183.972

1. Which digit is in the hundreds place?
2. What is the value of the digit 4?
3. In which place is the digit 7?
4. Which digit is in the thousandths place?
5. What is the value of the digit 8?

Drill Solutions

1. Which digit is in the hundreds place?

The hundreds place is three places to the left of the decimal point, so $\Longrightarrow$ 1 is in the hundreds place (3 is in the units place and 8 is in the tens place)

2. What is the value of the digit 4?

Since 4 is in the thousands place, its value is $\Longrightarrow$ $4 \times 1{,}000$ or 4,000

3. In which place is the digit 7?

Since 7 is two places to the right of the decimal point $\Longrightarrow$ 7 is in the hundredths place (9 is in the tenths place)

4. Which digit is in the thousandths place?

The thousandths place is three places to the right of the decimal point, so $\Longrightarrow$ 2 is in the thousandths place

5. What is the value of the digit 9?

Since 9 is in the tenths place, its value is $\Longrightarrow$ 9×0.1 or 0.9. This is equivalent to $\frac{9}{10}$ as well (we'll cover fractions in a later section).

Calculations

The ability to perform the four basic *operations* (addition, subtraction, multiplication, and division) is important for the test. While the GMAT isn't a test of your calculation skills, a solid level of arithmetic competence is expected.

Addition

The basic rule when performing addition by hand is to start *small*, with the right-hand side of each number you're adding, and then build up to the larger digits.

Start Small or Start Big: *Some shortcuts and estimation techniques start big rather than small. We'll look at these techniques in depth later.*

Example: Calculate 154 + 82

$$\begin{array}{r} 154 \\ +\ 82 \end{array}$$

Set the numbers vertically.

$$\begin{array}{r} 154 \\ \underline{+\ 82} \\ 6 \end{array}$$

Perform the addition one column at a time, starting with the rightmost column: 4 + 2 = 6.

$$\begin{array}{r} {}^{1} \\ 154 \\ \underline{+\ 82} \\ 36 \end{array}$$

Continue with the next column: 5 + 8 = 13.
Carry a 1 to the next column to the left when the current column adds to 10 or more.

Carry Over: *Why do you carry when a column adds to 10 or more? Since each column has only space for one digit, carrying allows you to account for the 10 without overflowing the space.*

$$\begin{array}{r} {}^{1} \\ 154 \\ \underline{+\ 82} \\ 236 \end{array}$$

Finish the addition: 1 + 1 = 2.

The solution or *sum* of the addition is 236.

Example: Calculate 0.562 + 8.35

$$\begin{array}{r} 0.562 \\ +\ 8.35 \end{array}$$

Set the numbers vertically so that the decimal places line up.

$$\begin{array}{r} 0.562 \\ +\ 8.350 \end{array}$$

Add a zero to the "missing" decimal place (or simply assume the blank space is zero).

Decimal Lineup: *When adding and subtracting decimals, you must line up the decimal points.*

$$\begin{array}{r} 0.562 \\ \underline{+\ 8.350} \\ 2 \end{array}$$

Perform the addition, starting from right to left: 2 + 0 = 2.

$$\begin{array}{r} {}^{1} \\ 0.562 \\ \underline{+\ 8.350} \\ 12 \end{array}$$

Continue with the next column: 6 + 5 = 11.
Carry a 1 to the next column to the left.

$$\begin{array}{r} {}^{1} \\ 0.562 \\ \underline{+\ 8.350} \\ 912 \end{array}$$

Continue the addition: 1 + 5 + 3 = 9.

$$\begin{array}{r} {}^{1} \\ 0.562 \\ \underline{+\ 8.350} \\ 8.912 \end{array}$$

Finish the addition: 0 + 8 = 8. Bring the decimal point straight down to the result.

The sum is 8.912.

Drill

Find the following sums:

1. 23 + 15
2. 876 + 47
3. 1.2 + 3.61
4. 34 + 18 + 69
5. 43.9 + 37.4 + 5.7

Drill Solutions

1. 23 + 15

Perform the addition:

$$\begin{array}{r} 23 \\ \underline{+15} \\ 38 \end{array}$$

Since there's no carrying, the addition is straightforward. If you're comfortable with mental math, this problem could safely be done in your head.

2. 876 + 47

Perform the addition, remembering to carry the 1 as necessary:

$$\begin{array}{r} {}^{1\,1} \\ 876 \\ \underline{+47} \\ 923 \end{array}$$

3. 1.2 + 3.61

Line up the decimal points, and perform the addition:

$$\begin{array}{r} 1.2 \\ \underline{+3.61} \\ 4.81 \end{array}$$

4. 34 + 18 + 69

Adding three (or more) numbers is essentially the same as adding two numbers; however, you may need to carry a number greater than 1:

$$\begin{array}{r} {}^{2} \\ 34 \\ +18 \\ \underline{+69} \\ 121 \end{array}$$

Since 4 + 8 + 9 = 21, write 1 for the units digit of the sum and carry the 2.

Note that since there are no more digits, when you add 2 + 3 + 1 + 6 = 12, you don't need to worry about carrying the 1 in 12.

Another way to solve is by adding two numbers first, and then adding the third number to the result. However, since this involves an extra step and extra time, it isn't ideal.

5. 43.9 + 37.4 + 5.7

Line up the decimal points, and perform the addition:

$$\begin{array}{r} {}^{1\,2} \\ 43.9 \\ +37.4 \\ \underline{+\ \ 5.7} \\ 87.0 \end{array}$$

Subtraction

Subtraction is the opposite of addition, and works in a somewhat similar fashion, starting with the right-hand side of the numbers.

Example: Calculate 627 − 354

$$\begin{array}{r} 627 \\ -354 \end{array}$$

Set the numbers vertically.

$$\begin{array}{r} 627 \\ \underline{-354} \\ 3 \end{array}$$

Perform the subtraction, starting from right to left: 7 − 4 = 3.

When you're subtracting a larger number from a smaller number (here, 2 − 5), you must *borrow* 1 from the column to the left.

> **Why Borrow?:** *Without borrowing, 2 − 5 results in −3, and there's no way to represent a negative in the result.*

$$\begin{array}{r} {\scriptstyle 5\,12} \\ \not{6}\not{2}7 \\ \underline{-354} \\ 3 \end{array}$$

To borrow, subtract 1 from the column to the left (6 − 1 = 5) and add 10 to the current column (2 + 10 = 12).

$$\begin{array}{r} {\scriptstyle 5\,12} \\ \not{6}\not{2}7 \\ \underline{-354} \\ 73 \end{array}$$

Continue the subtraction: 12 − 5 = 7.

$$\begin{array}{r} {\scriptstyle 5\,12} \\ \not{6}\not{2}7 \\ \underline{-354} \\ 273 \end{array}$$

Finish the subtraction: 5 − 3 = 2.

The result of a subtraction (in this case 273) is known as the *difference.*

Example: Calculate 400 − 37

$$\begin{array}{r} 400 \\ -37 \end{array}$$

Set the numbers vertically.

You must borrow to subtract 10 − 7 in the units column, but the tens digit of 400 is also 0. In effect, you need to borrow 1 from 40, reducing it to 39:

> **Zero Borrowing:** *Since you can't borrow from 0, you must borrow from the next digit to the left and turn the 0 into a 9.*

$$\begin{array}{r} {\scriptstyle 39\,10} \\ \not{4}\not{0}\not{0} \\ \underline{-37} \\ 3 \end{array}$$

Borrow 1 from the column to the left (40 − 1 = 39), and add 10 to the current column.
Perform the subtraction, starting from right to left: 10 − 3 = 7.

$$\begin{array}{r} {\scriptstyle 39\,10} \\ \not{4}\not{0}\not{0} \\ \underline{-37} \\ 63 \end{array}$$

Continue the subtraction: 9 − 3 = 6.

$$\begin{array}{r} {\scriptstyle 39\,10} \\ \not{4}\not{0}\not{0} \\ \underline{-37} \\ 363 \end{array}$$

Finish the subtraction: since there's nothing to subtract from 3, bring it straight down.

Example: Calculate 24.29 – 5.168.

Subtraction with Decimals:
As with addition, remember to line up the decimal points.

24.29 −5.168	Set the numbers vertically so that the decimal places line up.
24.290 −5.168	Add zeroes to any “missing” decimal places.
8 10 24.2~~90~~ −5.168 2	Since 8 is more than 0, borrow 1 from the column to the left (9 – 1 = 8) and add 10 to the current column (0 + 10 = 10). Subtract the rightmost column: 10 – 8 = 2.
8 10 24.2~~90~~ −5.168 22	Continue the subtraction: 8 – 6 = 2.
8 10 24.2~~90~~ −5.168 122	Continue the subtraction: 2 – 1 = 1.
1 14 8 10 ~~24~~.2~~90~~ −5.168 9122	Since 4 – 5 would be negative, borrow 1 from the column to the left (2 – 1 = 1), and add 10 to the current column (4 + 10 = 14). Continue the subtraction: 14 – 5 = 9.
1 14 8 10 ~~24~~.2~~90~~ −5.168 19.122	Since there's nothing to subtract from 1, bring it straight down to the result. Bring the decimal point straight down to the result.

Negative Numbers

What happens when you subtract a larger number from a smaller number?

Example: Calculate 24 – 35

In this case, the final result will be negative. However, it will be *less* negative than −35. To perform the calculation, **flip the terms** so that you subtract 24 from 35, then **make the result negative**:

35 – 24	Set the numbers vertically and perform the subtraction.
11 → −11	Turn into a negative value.

One Negative = Subtract:
Adding a negative is equivalent to subtracting a positive.

What if, instead of 24 – 35, you were asked to calculate 24 + (−35)? Adding a negative number is the same thing as subtracting the positive of that number. In either case, the result is −11. Likewise, −35 + 24 is also identical to the original operation.

Example: Calculate −13 – 5

Think about it: in this operation, you’re making a negative number *more* negative. In other words, it’s the same thing as adding 13 + 5 = 18, except the result will be negative: −13 – 5 = −18.

Finally, what if you subtract a negative number?

Example: Calculate $4 - -6$

Subtracting a negative is the same as adding a positive. So, this is equivalent to $4 + 6 = 10$.

Two Negatives = Add:
Subtracting a negative is equivalent to adding a positive.

Drill

Find the following differences (or sums):

1. 65 – 27
2. 63.37 – 17.52
3. 1,000 – 11.28
4. –28 – 53
5. 75 – –50
6. 4.6 – 12.3

Drill Solutions

1. 65 – 27

First, borrow from the tens to get 15 – 7 = 8 in the units column. Then, subtract the tens column to get 5 – 2 = 3 and finish the subtraction:

$$\begin{array}{r} {}^{5\,15} \\ \not{6}\not{5} \\ \underline{-27} \\ 38 \end{array}$$

2. 63.37 – 17.52

First, line up the decimals. Subtract the hundredths column to get 7 – 2 = 5. Then, borrow from the units to get 13 – 5 = 8 in the tenths column:

$$\begin{array}{r} {}^{2\ 13} \\ 6\not{3}.\not{3}7 \\ \underline{-17.52} \\ 85 \end{array}$$

Now, you must borrow from the tens digit to get 12 – 7 = 5, then finish the subtraction:

$$\begin{array}{r} {}^{5\,12\,13} \\ \not{6}\not{3}.\not{3}7 \\ \underline{-17.52} \\ 45.85 \end{array}$$

3. 1,000 – 11.28

Line up the decimals and add the "missing" zeroes. Then borrow 1 from 1000.0, reducing it to 999.9, before performing the subtraction:

$$\begin{array}{r} {}^{9\,9\,9\ 9\,10} \\ \not{1},\not{0}\not{0}\not{0}.\not{0}0 \\ \underline{-11.28} \\ 988.72 \end{array}$$

4. –28 – 53

Since here you're making a negative number more negative, add the two (positive) numbers, carrying as needed:

$$\begin{array}{r} {}^{1} \\ 28 \\ \underline{+53} \\ 81 \end{array} \Longrightarrow \text{The final result is } -81$$

5. 75 – –50

Remember, two negatives make a positive. This is equivalent to 75 + 50 :

$$\begin{array}{r} 75 \\ \underline{+50} \\ 125 \end{array}$$

6. 4.6 – 12.3

Here, you're subtracting a larger positive number from a smaller positive number. The result will be negative, but to calculate, flip the numbers, line up the decimals and perform the subtraction, borrowing as needed:

$$\begin{array}{r} {}^{1\ 13} \\ 1\not{2}.\not{3} \\ \underline{-4.6} \\ 7.7 \end{array} \Longrightarrow -7.7$$

Multiplication

Multiplication of single-digit numbers is performed using the times table from 0–9:

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Multiplication of numbers with more than one digit also depends on the times table, so this is an area where straight memorization is important. If you're shaky on the times table, set aside some time each study session to run through it.

Example: Calculate 6×8

Using the times table, $6 \times 8 = 48$.

Note that each of the following is an equivalent question:

Calculate $6 \cdot 8$ OR Calculate $6(8)$

× Looks Like *x*: *To indicate multiplication, use the dot or parentheses as a substitute for ×.*

Though the GMAT often uses the × symbol on its questions, it's a good idea for you to be comfortable using the dot and/or parentheses as well so that you can avoid confusion when the variable x is part of the problem.

Example: Calculate 34(82)

$$\begin{array}{r} 34 \\ \times\ 82 \end{array}$$

Set the numbers vertically.

$$\begin{array}{r} 34 \\ \underline{\times\ 82} \\ 8 \end{array}$$

You'll start by multiplying the rightmost digit in the bottom number by the rightmost digit in the top number ($2 \times 4 = 8$).

$$\begin{array}{r} 34 \\ \underline{\times\ 82} \\ 68 \end{array}$$

Next multiply the rightmost digit in the bottom number by the next digit to the left in the top number ($2 \times 3 = 6$).

Essentially, so far you've multiplied 2×34, resulting in 68. But you haven't yet touched the 8 from 82. Next you'll multiply that 8 times 34. But that 8 is actually "worth" 80. To account for this, place a zero in the next row:

$$\begin{array}{r} 34 \\ \underline{\times\ 82} \\ 68 \\ 0 \end{array}$$

The zero accounts for the fact that you are now multiplying by 80 instead of 8.

Place Zeroes: *To multiply multi-digit numbers, place one zero in the second row, two zeroes in the third row, etc.*

$$\begin{array}{r} {\scriptstyle 3} \\ 34 \\ \underline{\times\ 82} \\ 68 \\ 20 \end{array}$$

Multiply 8 × 4 = 32. But because 32 is greater than 10, write a 2 and carry the 3.

Carrying in multiplication serves the same purpose it did for addition. Since each column has only space for one digit, carrying allows you to account for the 3 in 32 without overflowing the space.

$$\begin{array}{r} {\scriptstyle 3} \\ 34 \\ \underline{\times\ 82} \\ 68 \\ 2720 \end{array}$$

Next, multiply 8 × 3 = 24. You must then add the carried 3 → 24 + 3 = 27. Write 27 to the left of the 2.

Note that since there are no more digits, you don't need to worry about carrying the 2 in 27.

$$\begin{array}{r} {\scriptstyle 3} \\ 34 \\ \underline{\times\ 82} \\ 68 \\ \underline{+2720} \\ 2788 \end{array}$$

The final step is adding the multiplied rows: the result is 2,788.

The result of a multiplication (in this case 2,788) is known as the *product*.

Example: Calculate 8.22 · 19.7

Multiplication with Decimals: *Begin by multiplying as usual—you'll address the decimal point at the end.*

$$\begin{array}{r} 8.22 \\ \times\ 19.7 \end{array}$$

Set the numbers vertically (the decimal places do not need to line up).

$$\begin{array}{r} {\scriptstyle 1} \\ 8.22 \\ \underline{\times\ 19.7} \\ 4 \end{array}$$

Start with the rightmost digit in the bottom number (the 7): 7 × 2 = 14. Write down 4, and carry the 1.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 8.22 \\ \underline{\times\ 19.7} \\ 54 \end{array}$$

7 × 2 is again 14, but you must also add the carried 1 → 14 + 1 = 15. Write down 5, and carry the 1.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 8.22 \\ \underline{\times\ 19.7} \\ 5754 \end{array}$$

Next, multiply 7 × 8 = 56 and add the carried 1 → 56 + 1 = 57. Write down 57.

$$\begin{array}{r} 8.22 \\ \underline{\times\ 19.7} \\ 5754 \\ 0 \end{array}$$

Place a zero in the next row. Erase or scratch out the carried numbers; you will have new carried numbers for this row.

For brevity's sake, I'll combine some steps for the rest of the problem.

```
  1 1
  8.22
× 19.7
  5754
 73980
```

Perform the multiplication for the 9 → 9 × 2 = 18. Carry the 1. Then 9 × 2 + 1 = 19. Carry the 1. Then 9 × 8 + 1 = 73.

```
  8.22
× 19.7
  5754
 73980
    00
```

Place *two* zeros in the next row. Erase or scratch out the carried numbers.

```
  8.22
× 19.7
  5754
 73980
 82200
```

Perform the multiplication for the 1. Since 1 times anything is itself, the results are 2, 2, and 8.

```
   8.22
 × 19.7
   5754
 +73980
 +82200
 161934
```

Add up the rows of multiplication results.

Multiplication with Decimals II: *Add up decimal places in the original numbers and move the decimal point over that many places in the product.*

Lastly, you must account for the decimals by adding up the number of places to the right of the decimal point. Since 8.22 has 2 places, and 19.7 has 1 place, there is a total of 3 places.

```
   8.22
 × 19.7
   5754
 +73980
 +82200
161.934
```

Move the decimal point over 3 places from the right.

As you can see, multiplication of three-digit numbers is tedious and prone to error. Though it's important to understand conceptually how to perform such operations, on the test, you'll want to look for shortcuts (such as estimation) to save yourself time. We'll go over some of those shortcuts in a moment.

Negative Numbers

Multiplying Signs: *If both quantities are the same sign, the result is positive. If they are different signs, the result is negative.*

It's also important to be aware of how negative numbers impact multiplication. **Positives and negatives are *always consistent* through multiplication:**

- A positive multiplied by another positive is always positive (e.g., $3 \times 5 = 15$)
- A negative multiplied by another negative is always positive (e.g., $-4 \times -9 = 36$)
- A positive multiplied by a negative OR a negative multiplied by a positive is always negative (e.g., $8 \times -2 = -16$)

Drill

Find the following products:

1. 8×23
2. $12(34)$
3. $0.96 \cdot 4.7$
4. $-34(22)$
5. -1.37×-42

Drill Solutions

1. 8×23

Set up the problem vertically and multiply, then add to find the final result:

$$\begin{array}{r} 8 \\ \underline{\times\ 23} \\ 24 \\ \underline{+160} \\ 184 \end{array}$$

2. $12(34)$

Set up the problem vertically and multiply, then add to find the final result:

$$\begin{array}{r} 12 \\ \underline{\times\ 34} \\ 48 \\ \underline{+360} \\ 408 \end{array}$$

3. $0.96 \cdot 4.7$

Set up the problem vertically and multiply, carrying the 4 from $6 \times 7 = 42$. Then, complete the multiplication for the first row (note that $0 \times 7 = 0$, but you don't need to write down the 0 since it will not affect the addition at the end):

$$\begin{array}{r} {\scriptstyle 4} \\ 0.96 \\ \underline{\times\ 4.7} \\ 672 \end{array}$$

Erase or scratch out the carried 4 to leave space for carried numbers for the next row. Continue multiplying, then add:

$$\begin{array}{r} {\scriptstyle 2} \\ 0.96 \\ \underline{\times\ 4.7} \\ 672 \\ \underline{+3840} \\ 4512 \end{array} \Longrightarrow \text{The final result is } 4.512$$

0.96 has 2 decimal places and 4.7 has 1 decimal place for a total of 3 places; thus, move the point over 3 places.

4. $-34(22)$

Since you're multiplying a negative and a positive, the product will be negative. Carry out the multiplication:

$$\begin{array}{r} -34 \\ \underline{\times\ 22} \\ 68 \\ \underline{+680} \\ 748 \end{array} \Longrightarrow -748$$

5. -1.37×-42

Since you're multiplying two negatives, the product will be positive. Set up the problem and multiply, carrying as needed:

$$\begin{array}{r} {\scriptstyle 1} \\ -1.37 \\ \underline{\times\ -42} \\ 274 \end{array}$$

Erase or scratch out the carried 1 to leave space for carried numbers for the next row. Continue multiplying, then add:

$$\begin{array}{r} {\scriptstyle 1\ \ 2} \\ -1.37 \\ \underline{\times\ -42} \\ 274 \\ \underline{+5480} \\ 5754 \end{array} \Longrightarrow 57.54$$

Since there are two decimal places, and you multiplied two negatives, the product is (positive) 57.54.

Division

Division is the opposite of multiplication, and like multiplication, makes use of the times table.

Example: Calculate $56 \div 8$.

Since $8 \times 7 = 56$, the result of $56 \div 8$ is 7. In division, the number you're dividing *into* (the 56) is known as the *dividend,* and the number you're dividing by (the 8) is known as the *divisor.* The result of the division (the 7) is known as the *quotient.*

Note that this question could also have been written in one of the following equivalent ways:

Calculate $\frac{56}{8}$ OR Calculate $8\overline{)56}$

The first way ($\frac{56}{8}$) is a *fraction*. Fractions are a whole topic unto themselves that we'll cover shortly (and go through in depth in Chapter 9: Fractions and Friends). The second way ($8\overline{)56}$) is known as a *division bracket* and can be helpful for calculations.

What happens when the number you're dividing into (the dividend) isn't on the times table?

Example: What is the quotient and remainder of $\frac{17}{3}$?

In this case, there is no whole number that results in 17 when multiplied by 3 because $5 \times 3 = 15$ and $6 \times 3 = 18$.

One way to handle this situation is with the concept of the *remainder*—the amount that is "left over" after you divide. Since $5 \times 3 = 15$, and $17 - 15 = 2$, you can say that $17 \div 3 = 5$ *remainder* 2.

To calculate division with larger numbers, use *long division.*

Example: Calculate the quotient and remainder of $362 \div 7$.

$7\overline{)362}$	Set up the division by placing the dividend (362) in the division bracket and the 7 to the left of the bracket.
$7\overline{)362}$	First, see if 7 will divide into the first digit (3) at least once. 7 is larger than 3, so it will not.
$7\overline{)362}$	Then, see if 7 will divide into the first *two* digits (36) at least once. 7 is smaller than 36, so it will.
$\begin{array}{r} 5 \\ 7\overline{)362} \end{array}$	**Divide** $36 \div 7$, which yields 5 *remainder* 1. Place the 5 above the 6, and ignore the remainder for the moment.
$\begin{array}{r} 5 \\ 7\overline{)362} \\ 35 \end{array}$	**Multiply** 7×5, which yields 35. Place the 35 below the 36.
$\begin{array}{r} 5 \\ 7\overline{)362} \\ \underline{-35} \\ 1 \end{array}$	**Subtract** $36 - 35$, which yields 1. Note that this is the same as the remainder of $36 \div 7$.

$$\begin{array}{r} 5 \\ 7\overline{)362} \\ \underline{-35} \\ 12 \end{array}$$

Bring Down the 2 from the dividend to create 12.

The bolded words (**Divide – Multiply – Subtract – Bring Down**) represent a process that you will repeat until you get to the end of the division.

Dad – Mom – Sister – Brother / Dog:
Use this mnemonic to help remember Divide – Multiply – Subtract – Bring Down.

$$\begin{array}{r} 51 \\ 7\overline{)362} \\ \underline{-35} \\ 12 \end{array}$$

Divide 12 ÷ 7, which yields 1 *remainder* 5. Place the 1 above the 2.

$$\begin{array}{r} 51 \\ 7\overline{)362} \\ \underline{-35} \\ 12 \\ 7 \end{array}$$

Multiply 7 × 1, which yields 7. Place the 7 below the 12.

$$\begin{array}{r} 51 \\ 7\overline{)362} \\ \underline{-35} \\ 12 \\ \underline{-7} \\ 5 \end{array}$$

Subtract 12 – 7, which yields 5 (the same as the remainder of 12 ÷ 7).

You've reached the end of the division, so there is nothing left to **Bring Down.** The final remainder is 5, so the result of 362 ÷ 7 is 51 *remainder* 5.

Let's see how long division works with decimals.

Example: Calculate the decimal value of 1.34 ÷ 0.4.

To do long division, your divisor (0.4 here) must be turned into an integer. To make it an integer, move the decimal point over the same number of places in both numbers:

Division with Decimals:
Turn the divisor into an integer, and remember to move the decimal point the same number of places in the dividend.

13.4 ÷ 4 — Move the decimal point one place to the right in both numbers so that your divisor becomes 4.

$$4\overline{)13.4}$$

Set up a division bracket. 4 won't divide into 1, but will divide into 13.

$$\begin{array}{r} 3 \\ 4\overline{)13.4} \end{array}$$

Divide 13 ÷ 4, which yields 3 *remainder* 1. Place the 3 above the 3 in the dividend (ignore the remainder for now).

$$\begin{array}{r} 3 \\ 4\overline{)13.4} \\ 12 \end{array}$$

Multiply 4 × 3, which yields 12. Place the 12 below the 13.

$$\begin{array}{r} 3 \\ 4\overline{)13.4} \\ \underline{-12} \\ 1 \end{array}$$

Subtract 13 – 12, which yields 1.

$$\begin{array}{r} 3 \\ 4\overline{)13.4} \\ \underline{-12} \\ 14 \end{array}$$

Bring Down the 4 to create 14.

```
   3.3
4)13.4
 −12
   14
    3.3
```

Divide 14 ÷ 4 → 3 *remainder* 2. Move the decimal point straight up. Place the 3 above the 4.

```
   3.3
4)13.4
 −12
   14
   12
```

Multiply 4 × 3, yielding 12. Place the 12 below the 14.

```
   3.3
4)13.4
 −12
   14
  −12
    2
```

Subtract 14 − 12, yielding 2.

Division with Decimals II: *If you run out of digits in the dividend, add zeroes after the decimal point as needed.*

At this point, you are out of digits in the dividend. Here, since you're calculating the decimal value rather than the quotient and remainder, you must continue. To keep going, add zeroes as needed.

```
   3.3
4)13.40
 −12
   14
  −12
    2
```

Since 13.4 is the same as 13.40, add a zero.

```
   3.3
4)13.40
 −12
   14
  −12
    20
```

Bring Down the 0 to create 20.

```
   3.35
4)13.40
 −12
   14
  −12
    20
```

Divide 20 ÷ 4, yielding 5 (with no remainder). Place the 5 above the 0.

Same Signs = Positive; Different Signs = Negative: *When multiplying or dividing terms, if both are the same sign (+ or –), the result is positive. If they are different signs, the result is negative.*

Since there's no further remainder, at this point you can stop. The result of 1.34 ÷ 0.4 is 3.35.

Like multiplication with large numbers, long division is tedious, so try to avoid it when possible. But it's important to understand it and to have it in your back pocket just in case.

Negative Numbers

Negative numbers impact division in exactly the same way that they do multiplication. **Positives and negatives are *always consistent* through division:**

- A positive divided by another positive is always positive (e.g., 15 ÷ 5 = 3)
- A negative divided by another negative is always positive (e.g., −36 ÷ −9 = 4)
- A positive divided by a negative OR a negative divided by a positive is always negative (e.g., −16 ÷ 2 = −8)

Drill

For questions 1–5, find the quotient and remainder.

1. $51 \div 6$
2. $\frac{49}{7}$
3. $\frac{127}{9}$
4. $14 \div 27$
5. $37 \div 15$

For questions 6–10, find the quotient including decimals.

6. $\frac{5}{8}$
7. $37 \div 5$
8. $7.82 \div 4$
9. $34 \div 0.02$
10. $21.024 \div 0.6$

Drill Solutions

1. $51 \div 6$

Since $6 \times 8 = 48$, the quotient is 8, and since $51 - 48 = 3$, the remainder is 3 $\Longrightarrow$ 8 *remainder* 3

2. $\frac{49}{7}$

Since $7 \times 7 = 49$, the quotient is 7, and there is no remainder $\Longrightarrow$ 7 (no remainder)

3. $\frac{127}{9}$

Use long division to solve:

$$\begin{array}{r} 14 \\ 9\overline{)127} \\ \underline{-9} \\ 37 \\ \underline{-36} \\ 1 \end{array}$$

$\Longrightarrow$ 14 *remainder* 1

4. $14 \div 27$

In this case, the divisor is larger than the dividend: 27 will not go into 14 at all (i.e., it will go in 0 times) $\Longrightarrow$ 0 *remainder* 14 If this isn't clear, see what the long division looks like:

$$\begin{array}{r} 0 \\ 27\overline{)14} \\ \underline{-0} \\ 14 \end{array}$$

Whenever the divisor is larger than the dividend, the quotient will be 0, and the remainder will be the dividend.

5. $37 \div 15$

Since $15 \times 2 = 30$, the quotient is 2, and since $37 - 30 = 7$, the remainder is 7 $\Longrightarrow$ 2 *remainder* 7

6. $\frac{5}{8}$

Perform the long division, adding zeroes after the decimal point as needed:

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

$\Longrightarrow$ 0.625

As we'll discuss in Chapter 9: Fractions & Friends, it's helpful to have simple fraction/decimal conversions like this memorized to avoid having to do the long division.

7. $37 \div 5$

Perform the long division, adding zeroes after the decimal point as needed:

$$\begin{array}{r} 7.4 \\ 5\overline{)37.0} \\ \underline{-35} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

$\Longrightarrow$ 7.4

Drill Solutions (continued)

8. 7.82 ÷ 4

Perform the long division, adding a zero after the last digit as needed:

```
    1.955
 4)7.820
  −4
    38
   −36
     22
    −20
      20
     −20
        0
```

⟹ 1.955

9. 34 ÷ 0.02

Move the decimal point over twice to the right for both numbers so that you're dividing by an integer: 3400 ÷ 2. Then divide, using long division if necessary:

```
    1700
 2)3400
  −2
    14
   −14
     0
```

Note that because 2 divided into 34 evenly (no remainder), the zeroes in 3400 can be simply moved up to the quotient.

⟹ 1700

10. 21.024 ÷ 0.6

Move the decimal point over once to the right for both numbers so that you're dividing by an integer: 210.24 ÷ 6. Then, perform the long division:

```
    35.04
 6)210.24
  −18
    30
   −30
     02
    −0
      24
     −24
       0
```

⟹ 35.04

Arithmetic Shortcuts

It's important to be comfortable performing the basic mathematical operations (addition, subtraction, multiplication, and division). However, because of the intense time pressure on the test, you'll want to calculate as quickly as possible using arithmetic shortcuts (often without any written work). Two great ways to shortcut your calculations are *using landmark values* and *rounding/estimation*.

Using Landmark Values

Landmark values are numbers that aren't exactly the numbers you're given, but are easy to work with and close enough to help. Generally, this means multiples of 10 and 100 (10, 20, 50, 100, 200, 700, etc.).

Landmark Values: *Multiples of 10 are easy to calculate using multiplication, addition, or subtraction.*

Example: Calculate 89 times 9

You could calculate using the standard method. But there's a quicker way:

$89 \times 10 - 89$ Instead of multiplying by 9, multiply by 10 and subtract 89.

$890 - 89 = 801$

In multiplying by 10, you "overshoot" the answer by one 89, meaning you need to subtract that 89 to get the right answer.

Similarly, when multiplying by 11 or 21, you can multiply by 10 or 20 instead to make your calculations easier.

Example: Calculate 43×21

$43 \times 20 + 43$ Instead of multiplying by 21, multiply by 20 and add the number.

$860 + 43 = 903$

In multiplying by 20, you "undershoot" the answer by one 43, meaning you need to add a 43 to get the right answer.

A similar multiplication shortcut comes into play when multiplying by 5. Since 5 is just half of 10, you can multiply by 10 and then cut the result in half.

Example: Calculate 864×5

$864 \times 10 \div 2$ Multiply the number by 10 and divide by 2.

$8640 \div 2 = 4320$

Landmark values also work for addition and subtraction.

Example: Calculate $1024 - 798$

$1024 - 800 + 2$ Instead of subtracting 798, subtract 800, and add 2 back.

$224 + 2 = 226$

Example: Calculate 38 + 92

Note that 38 is 2 below 40, and 92 is 2 above 90.

$40 - 2 + 90 + 2$	Replace 38 with 40 − 2 and 92 with 90 + 2.
$40 + 90 = 130$	The − 2 and + 2 cancel each other out.

Landmark values don't work quite as well for division (though rounding works well, as you'll soon see).

Drill

Use landmark values when possible to calculate the following:

1. 99×6
2. $51 \cdot 7$
3. 5×108
4. 32(11)
5. 4.8×4
6. $421 - 95$
7. $337 + 53$
8. $215 + 689$
9. $856 - 148$
10. $179 - 282$

Drill Solutions

1. 99×6

Since 99 is almost 100, calculate $100 \times 6 = 600$, and subtract one 6 → $600 - 6 \Longrightarrow 594$

2. $51 \cdot 7$

Since 51 is close to 50, calculate $50 \cdot 7 = 350$, and add one 7 $\Longrightarrow 357$

3. 5×108

Calculate $10 \times 108 = 1080$, and divide by 2 → $1080 \div 2 = 540$. Alternately, calculate $5 \times 100 = 500$, and add $5 \times 8 = 40$ → $500 + 40 \Longrightarrow 540$

4. 32(11)

Calculate $32(10) = 320$, and add 32 → $320 + 32 \Longrightarrow 352$

5. 4.8×4

Calculate $5 \times 4 = 20$, and then subtract $0.2 \times 4 = 0.8$ → $20 - 0.8 \Longrightarrow 19.2$

6. $421 - 95$

Calculate $421 - 100 = 321$, and then add 5 back in → $321 + 5 \Longrightarrow 326$

7. $337 + 53$

Note that 337 is 3 below 340, and 53 is 3 above 50, so they cancel out. Calculate $340 + 50 \Longrightarrow 390$

8. $215 + 689$

Note that 215 is 15 over 200, and 689 is 11 under 700 for a "net" of 4, so calculate $200 + 700 + 4 \Longrightarrow 904$

9. $856 - 148$

Since 148 is 2 less than 150, calculate $856 - 150 = 706$, and add 2 back in → $706 + 2 \Longrightarrow 708$

10. $179 - 282$

The result will be negative, so find $282 - 179$, and make the result negative. Calculate $282 - 180 = 102$, and add 1 back in → $102 + 1 = 103 \Longrightarrow -103$

Rounding and Estimation

Let's look again at a multiplication example you saw earlier.

Example: Calculate 8.22×19.7

Rounding simply means changing one number to another that's similar in value but easier to calculate. Here, you could *round to the nearest unit* by changing 8.22 to 8 and changing 19.7 to 20. Then, the calculation is one you may be able to do in your head: $8 \times 20 = 160$.

Let's say that the answer choices to this problem looked as follows:

(A) 123.872
(B) 144.093
(C) 161.934
(D) 173.982
(E) 188.509

Don't Calculate, Estimate:
Unless answer choices are closely spaced, look to estimate before performing tedious calculations.

Here, the answer choices are spread fairly far apart, indicating that your estimate gives you a high degree of confidence that the closest choice will be correct. Furthermore, since 8.22 is just above 8, and 19.7 is just below 20, you know that the "above" and the "below" cancel each other out somewhat. The answer has to be C. In many cases on the GMAT, close is close enough.

The rules for rounding are simple:

1. Decide on the digit you want to round.

2. Look at the digit immediately to the right.

3. If that digit is 5–9, round up. If that digit is 0–4, round down.

Example: Round 874.953 to the nearest hundredth, tenth, unit, ten, and hundred.

Depending on which digit you're rounding, you'll come up with different answers:

874.95	874.953 rounded to the nearest hundredth (3 means round down)
875.0	874.953 rounded to the nearest tenth (5 means round up)
875	874.953 rounded to the nearest unit (9 means round up)
870	874.953 rounded to the nearest ten (4 means round down)
900	874.953 rounded to the nearest hundred (7 means round up)

Which digit to round is sometimes a judgment call. Here, for example, if you needed to multiply by 6, it would be much easier to multiply 900×6 than 875×6. However, if the answer choices were relatively close together, you might still want to multiply 875×6 to get a more accurate estimate.

We'll go into more depth on estimation strategies in Chapter 12: Problem Solving 2.

Drill

1. Round 287.597 to the nearest whole number.

2. Round 6,499 to the nearest thousand.

3. Round 4.028 to the nearest hundredth.

For questions 4–6, estimate the totals by rounding to the nearest unit:

4. $9.95 + 8.99 + 4.90$

5. $32.6 - 14.398$

6. $56.4 \div 6.9$

Solve questions 7 and 8 using estimation:

7. 637×58

   (A) 25,653
   (B) 27,482
   (C) 29,666
   (D) 32,454
   (E) 36,946

8. $29.2 + 37.6 + 83.6$

   (A) 146.4
   (B) 150.4
   (C) 154.4
   (D) 158.4
   (E) 162.4

Drill Solutions

1. Round 287.597 to the nearest whole number.

Since the digit to the right of the decimal point is 5, round up $\Longrightarrow$ 288

2. Round 6,499 to the nearest thousand.

Since the hundreds digit is 4, round down $\Longrightarrow$ 6,000

3. Round 4.028 to the nearest hundredth.

Since the thousandth digit is 8, round up $\Longrightarrow$ 4.03

4. $9.95 + 8.99 + 4.90$

Round the numbers to 10, 9, and 5. Then, $10 + 9 + 5 \Longrightarrow 24$ (Note: The actual sum is 23.84.)

5. $32.6 - 14.398$

Rounded, the numbers are 33 and 14. Then, $33 - 14 \Longrightarrow 19$ (Note: The actual difference is 18.202.)

6. $56.4 \div 6.9$

Rounded, the numbers are 56 and 7. Then, $56 \div 7 \Longrightarrow 8$ (Note: The actual quotient is approximately 8.174.)

7. 637×58

   (A) 25,653
   (B) 27,482
   (C) 29,666
   (D) 32,454
   (E) 36,946

The answer choices are spread far apart, so estimation is a good strategy here. You can round 637 to 600 and 58 to 60. Then, $600 \times 60 = 36{,}000$. Since you rounded 58 up to 60 and 637 down to 600 (a proportionally bigger amount), you can expect that your estimate is on the low side $\Longrightarrow$ E is the only choice that makes sense and is correct

8. $29.2 + 37.6 + 83.6$

   (A) 146.4
   (B) 150.4
   (C) 154.4
   (D) 158.4
   (E) 162.4

Here, the answer choices are closer, so you must be careful. But note that if you round 29.2 up to 30 and 37.6 up to 40, and round 83.6 down to 80, the rounding up and rounding down will effectively cancel each other out. $30 + 40 + 80 = 150$ $\Longrightarrow$ B is the correct answer

Fractions and Percents

Fractions and percents are both important topics on the GMAT. Without a calculator, fractions are usually much easier to work with than decimals—and you don't get a calculator on the Quant section. And as you might expect on a business school entrance exam, you're often expected to calculate values like revenue and profits using percents.

For now, I want to make sure you're familiar with fraction and percent basics. (We'll go in depth in Chapter 9: Fractions & Friends.) In particular, it's important to be able to move easily between decimals, fractions, and percents, so we'll spend some time refreshing your conversion skills.

Fractions

Like decimals, *fractions* are used to represent non-integer values.

Example: Simplify $\frac{6}{9}$

The top portion of the fraction (the 6) is known as the *numerator,* and the bottom portion (the 9) is known as the *denominator.*

The basic rule when working with a fraction is: **You can multiply or divide by anything other than zero, as long as you do the same on both the top (numerator) and bottom (denominator).**

To simplify $\frac{6}{9}$, you need to put it in *lowest terms*. In this case, you can divide both numerator and denominator by 3, since 3 divides evenly into both 6 and 9:

$\frac{6 \div 3}{9 \div 3} \rightarrow \frac{2}{3}$

Since you can't divide by anything else without getting a decimal in the numerator or denominator, the fraction is in lowest terms.

Simplification: *To simplify a fraction (i.e., to put it in lowest terms), look for numbers that divide evenly into both numerator and denominator.*

Example: Represent 0.5 as a fraction.

It's often useful to *convert* fractions into decimals and vice versa. You probably know intuitively that $0.5 = \frac{1}{2}$, but let's go through the conversion process. Since 5 is in the tenths place, you can say that 0.5 is equal to five tenths:

$0.5 = \frac{5}{10}$

To put this fraction in lowest terms, divide both the top and bottom by 5:

$\frac{5 \div 5}{10 \div 5} \rightarrow \frac{1}{2}$

Example: Represent 1.03 as a fraction.

In this case, 3 is in the hundredths place, so:

$$1.03 = \frac{103}{100}$$

This fraction is already in lowest terms, since nothing divides into both 103 and 100 evenly.

However, since 1.03 = 1 + 0.03, you could also represent it as:

$$1 + 0.03 = 1 + \frac{3}{100} = 1\frac{3}{100}$$

Example: Represent $\frac{4}{5}$ as a decimal.

You may already have this memorized, but if you don't, one quick way to do the conversion is to multiply both numerator and denominator by 2:

$$\frac{4(2)}{5(2)} \rightarrow \frac{8}{10}$$

Then, simply place 8 in the tenths place of a decimal: 0.8.

Alternately, to represent $\frac{4}{5}$ as a decimal, you can perform long division:

$$\begin{array}{r} 0.8 \\ 5\overline{)4.0} \\ \underline{-0} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Sometimes, the division doesn't work out cleanly.

Example: Represent $\frac{1}{3}$ as a decimal.

Performing the division yields the following (to two decimal places):

$$\begin{array}{r} 0.33 \\ 3\overline{)1.00} \\ \underline{-0} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

Since there is still a remainder, you could keep going, but the pattern will repeat endlessly: 0.333333...

Some fractions have no *exact* decimal equivalents (at least, not that you can write out). In this case, you can say that $\frac{1}{3}$ is *approximately* 0.33, or $\frac{1}{3} \approx 0.33$.

Drill

1. Put $\frac{9}{12}$ in lowest terms.
2. Put $\frac{20}{35}$ in lowest terms.
3. Convert 0.6 to a fraction in lowest terms.
4. Convert 2.1 to a fraction.
5. Convert $\frac{5}{4}$ to a decimal.

Drill Solutions

1. Put $\frac{9}{12}$ in lowest terms.

You can divide both 9 and 12 by 3, resulting in $\Longrightarrow \frac{3}{4}$

2. Put $\frac{20}{35}$ in lowest terms.

You can divide both 20 and 35 by 5, resulting in $\Longrightarrow \frac{4}{7}$

3. Convert 0.6 to a fraction in lowest terms.

Since 6 is in the tenths place, 0.6 is equal to $\frac{6}{10}$. Divide the numerator and denominator by 2, resulting in $\Longrightarrow \frac{3}{5}$

4. Convert 2.1 to a fraction.

Since 1 is in the tenths place, 2.1 is equal to $\Longrightarrow \frac{21}{10}$ or $2\frac{1}{10}$

5. Convert $\frac{5}{4}$ to a decimal.

If you don't have the conversion memorized, you can use division to find it:

$$\begin{array}{r} 1.25 \\ 4\overline{)5.00} \\ \underline{-4} \\ 10 \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

$\Longrightarrow$ 1.25

Percents

Percent literally means "per hundred." So, the conversion from percent to fraction is pretty simple:

$$x\% = \frac{x}{100}$$

Example: Convert 23% to a fraction and to a decimal.

Percent Basics:
Percent = "per 100" and 100% = 1.

Since % means per 100, the conversion to a fraction is 23 per 100:

$$23\% = \frac{23}{100}$$

Of course, $\frac{23}{100}$ is equal to 0.23, so converting from a percent directly into a decimal is also fairly simple—just move the decimal point two places to the left:

$$23\% = 0.23$$

If you're ever confused, though, convert to a fraction first using $x\% = \frac{x}{100}$.

Example: Convert 0.015 to a percent.

To convert a decimal to a percent, you can use the fact that $100\% = \frac{100}{100} = 1$. So, multiplying any number by 100% is the same as multiplying by 1, which is always allowed since it doesn't change the value.

$$0.015 \cdot 100\%$$

Move the decimal point over twice to account for the two zeros in 100:

$$0.015 \cdot 100\% \rightarrow 1.5\%$$

As a shortcut, you can simply know that you must move the decimal point two places to the right to convert to a percent—but if you're ever confused, multiply by 100%.

Percent Of

One very common use of percents is taking a percentage *of* something. "Of" translates as *multiply*.

Math Translation:
"Of" translates as "×" or "times"; "is" translates as "=" or "equals."

Example: What is 50% of 12?

You can and should make use of common decimal / fraction conversions when working with percents.

Here, since $50\% = 0.5 = \frac{1}{2}$:

$$50\% \text{ of } 12 \rightarrow \frac{1}{2} \text{ of } 12 \rightarrow \frac{1}{2} \cdot 12 = 6$$

Example: What is 7% of 30?

In this case, 7% is not a common fraction, so you can simply convert to a decimal and multiply:

7% of 30 → 0.07×30

$$\begin{array}{r} 0.07 \\ \underline{\times\ 30} \\ 0 \\ \underline{+210} \\ 2.10 \end{array}$$

Perform the multiplication.

Move the decimal over two places.

7% of 30 is equal to 2.1.

Drill

1. Convert 15% to a fraction in lowest terms.
2. Convert 2.5 to a percent.
3. Convert $\frac{1}{5}$ to a percent.
4. What is 25% of 16?
5. What is 0.1% of 1000?

Drill Solutions

1. Convert 15% to a fraction in lowest terms.

15% is equivalent to $\frac{15}{100}$. Divide top and bottom by 5, resulting in $\Longrightarrow \frac{3}{20}$

2. Convert 2.5 to a percent.

To convert, multiply $2.5 \times 100\%$, which results in $\Longrightarrow$ 250%

3. Convert $\frac{1}{5}$ to a percent.

If you don't have the value memorized, long division will show you that $\frac{1}{5} = 0.2$. Then $0.2 \times 100\%$ is $\Longrightarrow$ 20%

4. What is 25% of 16?

25% is equivalent to $\frac{25}{100}$ or $\frac{1}{4}$. Then, $\frac{1}{4}$ of 16 is $\Longrightarrow$ 4

Alternately, multiply $0.25 \times 16 \Longrightarrow 4$

5. What is 0.1% of 1000?

0.1% is equivalent to 0.001. Then, 0.001×1000 is $\Longrightarrow$ 1

Expressions

Any way of representing a value is considered an *expression*. Expressions can be as simple as a single number or can consist of multiple items added, subtracted, multiplied, or divided.

Examples: All of the following are expressions:

$12 \qquad 2 \times 5 + 7 \qquad x \qquad \frac{45 - 0.001}{16} \qquad 10 \times |y| + 6 \div z \qquad x^2 - \sqrt{9}$

So far, we've dealt only with numbers, but you must also be familiar with *variables, exponents, roots, and absolute values*.

Variables

Variables are unknown quantities, often indicated with a letter such as x or y. Variables can be any real number, as long as the number fits the information given about that variable.

Sometimes information given about a variable allows you to *solve* for that variable.

Example: If $x - 5 = 0$, what is the value of x?

Since you were given this equation, it *must* hold true, and if it's true, x must be 5. In other words, 5 is the *solution* for x. A solution for a variable is a value that makes the equation true.

Exponents and Roots

Exponents are a shorthand way to indicate that a number is multiplied by itself a certain number of times.

Example: What is the value of 3^2?

3^2 indicates that 3 is multiplied by itself, appearing 2 times:

$$3^2 = 3 \times 3 = 9$$

Read 3^2 as "three to the second power" or "three squared." If you take a number to the third power, for example, 2^3, you can read it as "two to the third power" or "two cubed."

Example: If $x = 2$, what is the value of x^3?

x^3 indicates that x is multiplied by itself, appearing 3 times:

$$x^3 = x \times x \times x$$

So, if $x = 2$, then:

$$x^3 = 2^3 = 2 \times 2 \times 2 = 8$$

Powers higher than 3 work in a similar fashion (except that there is no equivalent to the terms "squared" or "cubed" for higher powers).

Square roots are like the second power (or squaring) in reverse: they indicate the number that, when multiplied by itself, equals the given value.

Example: What is the value of $\sqrt{4}$?

$\sqrt{4}$ indicates the number that, when multiplied by itself, equals 4:

$\sqrt{4} = 2$ Since $2 \times 2 = 4$

Note that there is another number that, when multiplied by itself, equals 4:

$-2 \times -2 = 4$

However, by convention, the radical sign ($\sqrt{\ }$) indicates the *positive* square root only.

Third roots and higher order roots are possible.

Example: What is the value of $\sqrt[3]{8}$?

$\sqrt[3]{8}$ indicates the third root of 8.

$\sqrt[3]{8} = 2$ Since $2 \times 2 \times 2 = 8$

There are additional rules regarding exponents and roots that will be covered in depth in Chapter 15: Exponents and Roots.

Absolute Value

Absolute value is denoted by two vertical bars (e.g., $|x|$) and is the value of a number without regard to its sign (positive or negative).

Absolute value for positive numbers is easy—it's just the number itself.

Example: $|7| \rightarrow 7$ The absolute value of 7 is 7.

The absolute value of a negative number is simply that number without the negative sign.

Example: $|-4| \rightarrow 4$ The absolute value of −4 is 4.

The absolute value of zero is zero.

Example: $|0| \rightarrow 0$

Drill

1. What is the value of 5^2?
2. What is the value of $(-3)^3$?
3. What is the value of $\sqrt{64}$?
4. If $x^3 = 27$, what is the value of x?
5. If $|x| = 14$, what could the value of x be?

Drill Solutions

1. What is the value of 5^2?

Since a number squared is that number times itself, $5^2 = 5 \times 5 \implies 25$

2. What is the value of $(-3)^3$?

A negative times a negative is positive, so $-3 \times -3 = 9$. Since the number is cubed, multiply again by $-3 \rightarrow 9 \times -3 \implies -27$

3. What is the value of $\sqrt{64}$?

Since $8 \times 8 = 64$, the value of $\sqrt{64}$ is $\implies 8$

4. If $x^3 = 27$, what is the value of x?

Since $3 \times 3 \times 3 = 27 \implies x$ must be 3

Note that x cannot be -3, since $-3 \times -3 \times -3 = -27$.

5. If $|x| = 14$, what could the value of x be?

The absolute value of either 14 or -14 is 14, so $\implies x$ could be either 14 or -14

Terms

A *term* is a simple expression that either contains no operations or has only multiplication, exponents and roots as operations (i.e., no addition, subtraction, or division). All of the following are terms.

Examples: 14 x $2x$ y^2 $3x^2y^3z^4$ $4\sqrt{x}$ $2\sqrt{3}$

Essentially, terms can either be numbers, variables, or a combination, with or without exponents or roots.

When numbers and variables are set next to each other, as in $2x$, multiplication is implied. In other words, $2x$ is equivalent to $2 \times x$. Likewise, $3x^2y^3z^4 = 3 \times x^2 \times y^3 \times z^4$.

Most importantly, you need to know how to *combine like terms*. Like terms are terms that have variables that look *exactly alike* (including any exponents or roots), and like terms can be added and subtracted.

Examples: $2x + 3x = 5x$ $4 - 1 = 3$

$3x^2y^3z^4 + 7x^2y^3z^4 = 10x^2y^3z^4$ $4\sqrt{x} - \sqrt{x} = 3\sqrt{x}$

Like Goes with Like:
Always look to add or subtract like terms—terms with the same variables, exponents, and roots.

In each case, the variables are the same (or nonexistent, as with $4 - 1 = 3$), and in each case, the numbers in front of the terms are added or subtracted. These numbers are known as *coefficients*. Any term without a number has an implied coefficient of 1 (since $1 \times$ anything is itself); thus, $4\sqrt{x} - \sqrt{x} = 4\sqrt{x} - 1\sqrt{x} = 3\sqrt{x}$.

How do terms relate to expressions? One term, by itself, is already a (simple) expression. *Unlike* terms added or subtracted make up more complex expressions.

Examples: $5x + 2y$ Two-term expression

$3x^2y^3z^4 + 4\sqrt{x} - 5$ Three-term expression

Note that these expressions cannot be simplified, since they have no like terms.

Example: Combine like terms in the following expression: $2 + 3x - 1 + 4x$.

Here, you have two sets of like terms: 2 and -1, and $3x$ and $4x$. The like terms aren't next to each other, but when dealing only with addition and subtraction, the order of the terms doesn't matter. You can combine $2 - 1$ and $3x + 4x$ to get $1 + 7x$.

Drill

Combine like terms in the following expressions.

1. $a + 5a - 3a$
2. $\sqrt{x} + \sqrt{xy} + \sqrt{y}$
3. $x^2 + 17x - 2x^2$
4. $1 + 5v + 9w - 11v + 4$
5. $x^2 + x + 1 - 2x^2 - 1 - x$

Drill Solutions

1. $a + 5a - 3a \rightarrow 3a$

All are like terms. Since $1 + 5 - 3 = 3$, the combined form is $\Longrightarrow 3a$

2. $\sqrt{x} + \sqrt{xy} + \sqrt{y}$

None of the terms are like, so it is already in combined form $\Longrightarrow \sqrt{x} + \sqrt{xy} + \sqrt{y}$

3. $x^2 + 17x - 2x^2$

Since x^2 and $-2x^2$ are like terms, they can be added to get $-x^2 \Longrightarrow -x^2 + 17x$

4. $1 + 5v + 9w - 11v + 4$

Combine $1 + 4 = 5$ and $5v - 11v = -6v \Longrightarrow 5 - 6v + 9w$

5. $x^2 + x + 1 - 2x^2 - 1 - x$

Combine $x^2 - 2x^2 = -x^2$, $x - x = 0$, and $1 - 1 = 0 \Longrightarrow -x^2$

Order of Operations and PEMDAS

Often, you'll need to *simplify* expressions: turn them into mathematically equivalent forms that are easier to grasp. When simplifying expressions, it's important to follow the rules of mathematics regarding the *order of operations*.

Example: Simplify $3 + 7 \times 2$

To simplify this expression, do you add the 3 and the 7 first, or multiply the 7 and the 2 first? This is important, because the two methods will give you different answers. If you add the 3 and 7 first, you get 10, which when multiplied by 2 gives you 20. If you multiply the 7 and the 2 first, you get 14, which when added to 3 gives you 17. So, is the answer 20 or 17?

The answer is 17, because mathematicians have decided that multiplication comes before addition. (The reason why isn't important; it's simply a universally agreed mathematical convention.) If you wanted to add the 3 and the 7 first, you'd need to use *parentheses* as in the following example.

Example: Simplify $(3 + 7) \times 2$

Mathematicians have decided that parentheses come before multiplication, so here, you would add the 3 and the 7 before multiplying the result by 2 (giving you a final answer of 20).

These rules regarding the order of operations are summarized with the acronym **PEMDAS**:

Parentheses
Exponents
Multiplication /
Division
Addition /
Subtraction

Please Excuse My Dear Aunt Sally: *Use this mnemonic to help remember* **PEMDAS.**

Parentheses come first, then exponents, then multiplication AND division (from left to right), and finally, addition AND subtraction (from left to right).

Example: Simplify $16 + 2(7 - 5) \div 2^2$

$16 + 2(2) \div 2^2$	First, solve within the **P**arentheses.
$16 + 2(2) \div 4$	Replace the **E**xponent term.
$16 + 4 \div 4$	Perform the first **M**ultiplication / **D**ivision from left to right.
$16 + 1$	Perform the second **M**ultiplication / **D**ivision from left to right.
17	Perform the **A**ddition / **S**ubtraction.

Note that 2(2) is equivalent to 2×2. As you saw earlier, anything set right next to parentheses simply means that value is multiplied by what's inside the parentheses.

Simplification is a common task on the GMAT; you are much more likely to see 17 as an answer choice than $16 + 2(7 - 5) \div 2^2$ (although it is possible).

Think Simple: *On the GMAT, even if the question doesn't ask you explicitly to simplify, you should usually do so anyway—simplifying makes calculations easier, and answers are likely to be in simplified form.*

The Division Bar and Absolute Value

As you saw in the Division section, rather than using the ÷ symbol for division, often expressions will contain a division bar to indicate division.

Example: $\frac{55 - 23}{6 + 2}$

The key thing to remember with a division bar is to treat everything above the bar (the numerator) as though it were in parentheses, and everything below the bar (the denominator) as though it were in parentheses:

$$\frac{55 - 23}{6 + 2} = \frac{(55 - 23)}{(6 + 2)} = (55 - 23) \div (6 + 2)$$

The above shows three ways of saying the same thing. To solve, follow the normal rules of PEMDAS:

$\frac{55 - 23}{6 + 2}$	Given expression.
$\frac{32}{8}$	Work above the bar and below the bar separately at first.
4	Perform the division.

Division Bar and Absolute Value = Parentheses: *For the order of operations, they are equivalent to the "P" in PEMDAS.*

Similar to the division bar, absolute value functions like parentheses in determining order of operations.

Example: Simplify $42 + |13 - 15| - (1 - 2)$

$42 + \lvert -2 \rvert - (-1)$	Simplify within the parentheses / absolute value first.
$42 + 2 - (-1)$	The absolute value of −2 is 2.
$42 + 2 + 1$	Subtracting −1 is the same as adding 1.
45	Simplified expression.

Drill

Simplify the following expressions.

1. $18 + 4(3 - 7)^2$
2. $x(y + 2y) + 4x$
3. $\frac{19 + 2^3}{\sqrt{9}}$
4. $\frac{|2 - 2 \times 4|}{3^2}$
5. $(3 + |3 - 4^2|)y$
6. $2x - (3 - 5^2)x$

Drill Solutions

1. $18 + 4(3 - 7)^2$

Following PEMDAS, first simplify within the parentheses: $18 + 4(-4)^2$

Next, do the exponent. Since $-4 \times -4 = 16$, the expression becomes: $18 + 4(16)$

Multiply $4(16) = 64$, giving you: $18 + 64$

Finally, add to get $\Longrightarrow 82$

2. $x(y + 2y) + 4x$

First, simplify within the parentheses by combining like terms: $x(3y) + 4x$

Multiply x by $3y$ to get $\Longrightarrow 3xy + 4x$

These are not like terms, so they can't be combined.

3. $\frac{19 + 2^3}{\sqrt{9}}$

First, you must simplify the numerator and denominator separately (each is equivalent to parentheses).

Simplify 2^3 to 8 and $\sqrt{9}$ to 3 to get: $\frac{19 + 8}{3}$

Add $19 + 8 = 27$, giving you: $\frac{27}{3}$

Divide 27 by 3 to get $\Longrightarrow 9$

4. $\frac{|2 - 2 \times 4|}{3^2}$

First, you must simplify the numerator and denominator separately.

In the numerator, $2 \times 4 = 8$, then $2 - 8 = -6$. In the denominator, $3^2 = 9$. You now have: $\frac{|-6|}{9}$

Since $|-6| = 6$, you're left with $\frac{6}{9}$. Divide both numerator and denominator by 3; this reduces to $\Longrightarrow \frac{2}{3}$

5. $(3 + |3 - 4^2|)y$

Here, you have an absolute value within parentheses. Work on the inner expression (the absolute value) first.

$4^2 = 16$, and $3 - 16 = -13$. Since $|-13| = 13$, you now have: $(3 + 13)y$

Add within the parentheses: $3 + 13 = 16$. Finally, $(16)y$ is equivalent to $\Longrightarrow 16y$

6. $2x - (3 - 5^2)x$

First, work within the parentheses. Since $5^2 = 25$ and $3 - 25 = -22$, you have: $2x - (-22)x$

This is equivalent to $2x - -22x$. Two negatives make a positive, so you get: $2x + 22x$

Combine like terms: $2x + 22x \Longrightarrow 24x$

The Distributive Property

There's a special rule, known as the *distributive property,* that applies when a term is multiplied by a parenthetical expression. The distributive property allows you to "go around" PEMDAS in these situations by multiplying the term outside the parenthetical by each term inside it.

Example: Simplify $2(x + 3)$

Since x and 3 are unlike terms, they can't be combined. To simplify, you must *distribute* the 2 to the x and to the 3:

$$2(x + 3) \rightarrow 2 \times x + 2 \times 3 \rightarrow 2x + 6$$

The distributive property also works if the outside term has a variable.

Example: Simplify $3x(2y + 4x)$

$$3x(2y + 4x) \rightarrow 3x \times 2y + 3x \times 4x \rightarrow 6xy + 12x^2$$

You can also use the distributive property with division.

Example: Simplify $\frac{6x + 4}{2}$

In this case, distribute the division by 2 to both $6x$ and 4:

$$\frac{6x + 4}{2} \rightarrow \frac{6x}{2} + \frac{4}{2} \rightarrow 3x + 2$$

Using the distributive property (and its inverse, factoring) is often key to cracking tough algebra setups. We'll go further into these concepts in Chapter 16: Expressions & Equations; for now, make sure you're comfortable with the basics of distributing.

Drill

Simplify the following expressions:

1. $5(x - 6)$
2. $(7 - y)2x$
3. $x(x + 1)$
4. $\frac{-2(m + 7)}{3(n - 1.5)}$
5. $-7(x + y - z)$
6. $-a(3a + 2 - 7a) + a(1 - 2a)$
7. $\frac{3x + 9y}{3}$
8. $\frac{4x + 8y + 12z}{4}$

Drill Solutions

1. $5(x - 6)$

Distribute the 5 to the terms inside the parentheses: $5 \times x - 5 \times 6 \implies 5x - 30$

2. $(7 - y)2x$

Distribute the $2x$ to the terms inside the parentheses: $2x \times 7 - 2x \times y \implies 14x - 2xy$

3. $x(x + 1)$

Distribute the x to the terms inside the parentheses: $x \times x + x \times 1 \implies x^2 + x$

4. $\frac{-2(m + 7)}{3(n - 1.5)}$

In the numerator, $-2 \times m - 2 \times 7 \rightarrow -2m - 14$

In the denominator, $3 \times n - 3 \times 1.5 \rightarrow 3n - 4.5$

Together, you have $\implies \frac{-2m - 14}{3n - 4.5}$

5. $-7(x + y - z)$

Distribute the -7 to the terms inside the parentheses: $-7 \times x - 7 \times y - 7 \times -z \implies -7x - 7y + 7z$

6. $-a(3a + 2 - 7a) + a(1 - 2a)$

Before distributing, it's more efficient to combine like terms in the first parentheses: $-4a + 2$

Then, distribute the $-a$ through the first parentheses: $-a(-4a + 2) \rightarrow -a \times -4a - a \times 2 \rightarrow 4a^2 - 2a$

Distribute the a through the second parentheses: $a \times 1 + a \times -2a \rightarrow a - 2a^2$

Now, you have: $4a^2 - 2a + a - 2a^2$. Combine like terms to get $\implies 2a^2 - a$

7. $\frac{3x + 9y}{3}$

Distribute the division by 3 to $3x$ and $9y$ $\rightarrow \frac{3x}{3} + \frac{9y}{3} \implies x + 3y$

8. $\frac{4x + 8y + 12z}{4}$

Distribute the division by 4 to the terms in the numerator:

$\frac{4x}{4} + \frac{8y}{4} + \frac{12z}{4} \implies x + 2y + 3z$

Equations and Inequalities

Equations set one value *equal* to another. In other words, they tell you that one expression has the same value as another. Equations are very useful when you have one or more variables and need to find their solutions.

The basic rule when working with equations is that you must **do the same thing to both sides.** Let's see this first with a simple example.

Example: If $x - 3 = 0$, what is the value of x?

$x - 3 + 3 = 0 + 3$	Add 3 to both sides.
$x + 0 = 0 + 3$	The −3 and 3 on the left side of the equation add up to 0.
$x = 3$	Final form of the equation.

By adding 3 to both sides, you keep the equation "balanced." Though the expressions on both sides have changed value, they are still *equal* to each other.

Treat Both Sides Equally:
Do the same thing to both sides of an equation.

The above is a simple example of a common process you'll use with equations, known as *isolating the variable*: leaving the variable by itself on one side of the equation. Note that since 3 was subtracted on the left side, you "undid" the subtraction by adding 3. Addition and subtraction "undo" each other, and multiplication and division "undo" each other.

Example: $\frac{y - 1}{16(3 - 1)} = \frac{75}{32}$

In order to solve this equation, you'll want to isolate the y on the left side of the equation. To make your life easier, let's first simplify the expression on the left side:

$\frac{y - 1}{16(3 - 1)}$	Expression on left side of the equation.
$\frac{y - 1}{16(2)}$	First, simplify within the parentheses.
$\frac{y - 1}{32}$	Then, perform the multiplication.

Now that the left side has been simplified, you can look at the equation as a whole and begin to isolate y:

$\frac{y - 1}{32} = \frac{75}{32}$	
$\frac{y - 1}{32} \times 32 = \frac{75}{32} \times 32$	To "undo" the division by 32, multiply both sides by 32.
$y - 1 = 75$	Now, you just need to "undo" the subtraction by 1.
$y - 1 + 1 = 75 + 1$	Add 1 to both sides.
$y = 76$	y is now isolated and solved.

Isolate the Variable:
Your goal with an equation is usually to get a variable by itself.

Both of the preceding equations are examples of *linear equations*. Linear equations are equations where any variables are only taken to the first power (i.e., the variables are not squared, cubed, etc.). With limited exceptions, when a linear equation contains only one variable, it's possible to solve for that variable by isolating it.

Systems of Equations and Substitution

Let's say that you were given *two* related equations (known as a *system* of equations) and asked to solve for x, as in the following example.

Example: If $\frac{y-1}{16(3-1)} = \frac{75}{32}$ and $y = 2x - 10$, what is the value of x?

As you just saw, you can solve the first equation for y, telling you that $y = 76$. Now, you can plug in, or *substitute*, 76 for y in the second equation:

$76 = 2x - 10$	Replace y with 76.
$86 = 2x$	Add 10 to both sides.
$43 = x$	Divide both sides by 2.

Note here that, because you are given $y = 2x - 10$, you could also have substituted before even solving the first equation:

$\frac{y-1}{16(3-1)} = \frac{75}{32}$	The first given equation.
$\frac{(2x-10)-1}{16(3-1)} = \frac{75}{32}$	Replace y with $2x - 10$.
$\frac{2x-11}{16(3-1)} = \frac{75}{32}$	Simplify the left-side numerator to $2x - 11$.
$\frac{2x-11}{32} = \frac{75}{32}$	Simplify the left-side denominator to 32.
$2x - 11 = 75$	Multiply both sides by 32.
$2x = 86$	Add 11 to both sides.
$x = 43$	Divide both sides by 2.

Subbing In: *Whenever two quantities are equal, you can substitute one for the other in any equation or expression.*

As is often the case in math, there's more than one way to solve. As long as you follow the rules, you can determine the solution path you want to take.

Drill

Solve the following equations for the indicated variable:

1. If $4(x + 1) = 12$, what is the value of x?
2. If $\frac{3m}{4} = 2m$, what is the value of m?
3. If $\frac{x+2}{3} = x - 4$, what is the value of x?
4. If $6 = \frac{y-1}{-2y}$, what is the value of y?

Solve the following systems of equations for both variables:

5. $x = 2y + 4$ and $y = 8$
6. $3(a + 2) = 2b$ and $2a = 6$
7. $2\left(\frac{1}{2}r - 3\right) = s$ and $s = 3r$
8. $3y + 2 = 5$ and $x = \frac{y-1}{4}$

Drill Solutions

1. If $4(x + 1) = 12$, what is the value of x?

$x + 1 = 3$ Divide both sides of the equation by 4.

Then, subtract 1 from both sides $\Longrightarrow x = 2$

2. If $\frac{3m}{4} = 2m$, what is the value of m?

$3m = 8m$ Multiply both sides of the equation by 4.

$0 = 5m$ Subtract $3m$ from both sides.

Then, divide both sides by 5 $\Longrightarrow m = 0$

3. If $\frac{x+2}{3} = x - 4$, what is the value of x?

$x + 2 = 3(x - 4)$ Multiply both sides by 3.

$x + 2 = 3x - 12$ Distribute the 3 to the x and the -4 on the right side.

$2 = 2x - 12$ Subtract x from both sides.

$14 = 2x$ Add 12 to both sides.

Finally, divide both sides by 2 $\Longrightarrow 7 = x$

4. If $6 = \frac{y-1}{-2y}$, what is the value of y?

$-12y = y - 1$ Multiply both sides by $-2y$.

$0 = 13y - 1$ Add $12y$ to both sides.

$1 = 13y$ Add 1 to both sides.

Finally, divide both sides by 13 $\Longrightarrow \frac{1}{13} = y$

5. $x = 2y + 4$ and $y = 8$

$x = 2(8) + 4$ Substitute $y = 8$ into the first equation.

$x = 16 + 4 = 20$ Multiply and add.

Thus $\Longrightarrow x = 20$ and $y = 8$

6. $3(a + 2) = 2b$ and $2a = 6$

As is often the case with systems of equations, there is more than one way to solve. Do whatever seems easiest.

Since the second equation is simpler, let's start there by dividing by 2 $\rightarrow a = 3$

$3(3 + 2) = 2b$ Substitute $a = 3$ into the 1st equation.

$3(5) = 2b$ Add within the parentheses.

$15 = 2b$ Multiply on the left side.

$\frac{15}{2} = b$ Divide both sides by 2.

Thus $\Longrightarrow a = 3$ and $b = \frac{15}{2}$ (or 7.5)

7. $2\left(\frac{1}{2}r - 3\right) = s$ and $s = 3r$

$2\left(\frac{1}{2}r - 3\right) = 3r$ Substitute $s = 3r$ into the first equation.

$2 \times \frac{1}{2}r - 2 \times 3 = 3r$ Distribute the 2 to $\frac{1}{2}r$ and -3.

$r - 6 = 3r$ Simplify.

$-6 = 2r \rightarrow -3 = r$ Subtract r and divide by 2 to solve.

Now, plug $r = -3$ back into the second equation $\rightarrow s = 3(-3) \rightarrow s = -9$

Thus $\Longrightarrow r = -3$ and $s = -9$

8. $3y + 2 = 5$ and $x = \frac{y-1}{4}$

Let's start with the first equation:

$3y = 3$ Subtract 2 from both sides.

$y = 1$ Divide both sides by 3.

$x = \frac{1-1}{4}$ Substitute $y = 1$ into the second equation.

$x = \frac{0}{4} = 0$ Simplify.

Thus $\Longrightarrow x = 0$ and $y = 1$

Inequalities

The following are considered inequalities:

$>$	Greater than
$<$	Less than
$\geq$	Greater than or equal to
$\leq$	Less than or equal to

In working with inequalities, there are a couple of key rules to remember. The first is:

1. You can do the same things to an inequality that you can do to an equation.

That is, you can add, subtract, multiply, or divide the two sides of the inequality by anything, as long as it's the same thing on both sides.

Example: If $x + 2 < 9$, solve for x.

To solve, isolate the variable by subtracting 2 from both sides, just as you would with an equation. Thus, $x < 7$.

2. If you multiply or divide both sides by a negative, you must switch the inequality sign.

Negative Flip:
Remember to flip the inequality sign when you multiply or divide by a negative.

To prove this rule, let's look at a simple example.

Example: $7 > 4$

This is a true statement (read as "7 is greater than 4"). Now, multiply both sides of the inequality by -1:

$-7 < -4$ For the inequality still to be true, you must flip the sign.

While the rule is straightforward, the GMAT can test it in interesting ways. It's important to stay alert to this rule whenever you work with inequalities.

Example: If $-5x \leq 15$, what is the range of possible values of x?

Again, you must isolate x, just as you would with an equation.

$\frac{-5x}{-5} \geq \frac{15}{-5}$ Divide both sides by -5, remembering to flip the inequality sign.

$x \geq -3$ x could be any value greater than or equal to -3.

Drill

1. How many integers x satisfy both $7 \leq x < 9$ and $8 \leq x \leq 11$?

2. If $2g > 4$, what is the range of possible values of g?

3. If $-3x \leq 9$, what is the range of possible values of x?

4. If $\frac{2x + 4}{-2} > 5x$, what is the range of possible values of x?

5. If $t \geq 7$ and $2t = 6s - 10$, what is the range of possible values of s?

Drill Solutions

1. How many integers x satisfy both $7 \leq x < 9$ and $8 \leq x \leq 11$?

7 and 8 are the only integers that are greater than or equal to 7 and less than 9 (the first inequality). Of 7 and 8, only 8 is greater than or equal to 8 and less than or equal to 11 (the second inequality) $\Longrightarrow$ 1 integer (8)

2. If $2g > 4$, what is the range of possible values of g?

Divide both sides by 2 $\Longrightarrow g > 2$

In other words, g can be any value greater than 2.

3. If $-3x \leq 9$, what is the range of possible values of x?

Divide both sides by −3, remembering to flip the inequality sign $\Longrightarrow x \geq -3$

4. If $\frac{2x+4}{-2} > 5x$, what is the range of possible values of x?

Multiply both sides by −2, remembering to flip the inequality sign → $2x + 4 < -10x$

Add $10x$ and subtract 4 from both sides → $12x < -4$

Divide both sides by 12 → $x < -\frac{4}{12}$

Simplify $-\frac{4}{12}$ to $-\frac{1}{3} \Longrightarrow x < -\frac{1}{3}$

5. If $t \geq 7$ and $2t = 6s - 10$, what is the range of possible values of s?

You want to isolate t in the equation so you can substitute it into $t \geq 7$. (You CANNOT substitute $t \geq 7$ in for t in the equation—inequalities can never be substituted this way.)

Divide both sides by 2 → $t = \frac{6s-10}{2}$

Distribute the division by 2 on the right side → $t = \frac{6s}{2} - \frac{10}{2}$ → $t = 3s - 5$

Now, substitute into $t \geq 7$ → $3s - 5 \geq 7$

Add 5 to both sides → $3s \geq 12$

Divide both sides by 3 $\Longrightarrow s \geq 4$

Introduction to Word Problems

One way the GMAT can add complexity to a question is by making it a word problem. Word problems tend to be more difficult than straight math problems because there are many opportunities for mistakes when translating the words into math.

Example: Jeff weighs 60 pounds less than twice Eduardo's weight.

This can be translated into algebra as:

$j = 2e - 60$

In translating from English to algebra, there are a few key principles to remember:

1. **Use *letters or words* as variables to stand in for unknowns.**
2. **Look for *equal quantities* that you can turn into equations.**
3. **Look for *keywords / key phrases* to translate.**
4. ***Translate the question* to focus your solving.**

Let's dig deeper into each of these principles.

1. Use *letters or words* as variables to stand in for unknowns.

In the example, setting j as Jeff's weight and e as Eduardo's weight is the simplest way to represent these unknown quantities. But be smart—if Eduardo were named John instead, you might want to write out their names to distinguish the two weights:

Choose Wisely: *Pick letters or words that clearly refer to what they stand for—that way you won't get confused.*

$Jeff = 2 \cdot John - 60$

Alternately, you could write down $x = Jeff$ and $y = John$ and then say:

$x = 2y - 60$

This is a good move, especially if you expect to do a fair amount of algebraic manipulation in the rest of the problem, so that you don't have to keep rewriting their names. Just make sure that you write down who's represented by each variable!

2. Look for *equal quantities* that you can turn into equations.

In the example sentence, the word "equals" does not appear. However, it's clear that the quantities "Jeff's weight" and "60 pounds less than twice Eduardo's weight" are meant to be equal.

Not every word problem will translate into equations, but most will—especially the more difficult problems to decipher. For this reason, pay attention to any quantities that are or can be equal, and use them to form equations.

3. Look for *keywords / key phrases* to translate.

In the example problem, "less than" and "twice" indicate subtraction and multiplication, respectively. This may seem fairly straightforward—just remember that "less than" means "subtracted from" and not "minus." It's easy to miss this and end up with $j = 60 - 2e$, which is incorrect!

Keywords also exist to indicate "equals." However, as you saw in the example, sometimes there is no keyword for "equals," and in that case, you just need to look for equal quantities.

Here are some common terms and how they translate into math:

+	−	×	÷	=
more than *increased by* *sum* *total* *together* *plus* *buys*	*fewer than* *less than* *decreased by* *reduced by* *differ from* *difference* *minus* *sells*	*times* *twice* *double* *triple* *product* *of*	*per* *ratio* *quotient* *for every*	*is / was* *are / were* *has / have* *same* *equivalent* *identical* *represents* *consists of* *accounts for* *as many as* *expressed as*

>	<	≥	≤	**Variable**
more than *greater than*	*fewer than* *less than*	*at least* *no less than* *minimum*	*at most* *no more than* *maximum*	*what number* *what amount* *what value* *how many*

To translate correctly, make sure you understand the described scenario. For example, "more than" could indicate addition or an inequality depending on context.

Here are a few examples of translations that sometimes trip people up:

- "3 fewer than x" / "3 less than x": Translate as "$x - 3$" NOT as "$3 - x$"
- "Anirban sold twice as many cars as Jeremy": Translate as "$a = 2j$" NOT as "$2a = j$"
- "x is greater than $9y$": Translate as "$x > 9y$" NOT as "$9y + x$"

4. *Translate the question* to focus your solving.

With word problems, it's often smart to restate the question so that you're clear on what you're solving for. If possible, you should translate the question into algebraic form.

Know What's Asked of You: *Being clear on what you're solving for can save you from unnecessary calculations, especially on more difficult word problems.*

Let's see this in practice by extending the example.

Example: Jeff weighs 60 pounds less than twice Eduardo's weight. Eduardo weighs 90 pounds less than twice Jeff's weight. How much does Jeff weigh?

Translated, this looks like:

$j = 2e - 60$

$e = 2j - 90$

What is j? The question in algebraic form.

To solve, should you substitute for j or e? Since the question asks for j, it's more efficient to substitute for e:

$j = 2(2j - 90) - 60$	Replace e in the first equation with $2j - 90$.
$j = 4j - 180 - 60$	Distribute the 2 through the parentheses.
$3j = 240$	Combine like terms: subtract j from both sides and add 240 to both sides.
$j = 80$	Divide both sides by 3 to solve.

Though you could also solve for e, there's no need to—you've already answered the question.

Algebraic translation is a core skill to master, since so many GMAT Quant and Data Insights questions come in the form of word problems.

Drill

1. Suki owns a number of shares of stock. If she were to sell half of her shares and then buy 25 shares, she would own 175 shares of stock. How many shares of stock does she currently own?

2. Anthony has five marbles. Leah has three more than four times as many marbles as Anthony. How many marbles does Leah have?

3. Between them, Holden and Katya have 14 coins. Holden has 2 more coins than Katya. How many coins do each of them have?

4. Morristown has 16,000 residents. If its number of residents decreases by 2,000, Morristown will still have more than twice as many residents as Janesville. What is the range of possible numbers of residents Janesville has?

5. Barry's Bus Company has twice as many buses as its nearest competitor, Flora's Fleet Inc. If Flora's Fleet Inc. acquired 3 more buses, it would have one less than Barry's Bus Company. How many buses does Barry's Bus Company have?

Drill Solutions

1. Suki owns a number of shares of stock. If she were to sell half of her shares and then buy 25 shares, she would own 175 shares of stock. How many shares of stock does she currently own?

Let's designate s as Suki's current shares of stock.

If she sells half her shares, her number of shares would be divided in two: $\frac{s}{2}$

If she then buys 25 shares, her number of shares would increase by 25: $\frac{s}{2} + 25$

Now, this number of shares would be equivalent to 175 shares, so you can translate this as an equation: $\frac{s}{2} + 25 = 175$

The question is simply: What is s?

Now, solve the equation for s:

$\frac{s}{2} = 150$ Subtract 25 from both sides.

$s = 300$ Multiply both sides by 2.

$\Longrightarrow$ Suki currently owns 300 shares of stock

2. Anthony has five marbles. Leah has three more than four times as many marbles as Anthony. How many marbles does Leah have?

Let's use a for Anthony's marbles and L for Leah's marbles.

You can translate the first sentence as: $a = 5$

You can translate the second sentence as: $L = 4a + 3$

You can translate the question as: What is L?

Then solve. Substitute $a = 5$ into the second equation and calculate: $L = 4(5) + 3 \rightarrow L = 23 \Longrightarrow$ Leah has 23 marbles

3. Between them, Holden and Katya have 14 coins. Holden has 2 more coins than Katya. How many coins do each of them have?

Translate the first sentence as: $h + k = 14$

Translate the second sentence as: $h = k + 2$

Then, the question is: What is h and what is k?

It's probably easiest to substitute $h = k + 2$ into the first equation and simplify: $(k + 2) + k = 14 \rightarrow 2k + 2 = 14$

Subtract 2 from both sides, and divide by 2 to isolate k: $2k = 12 \rightarrow k = 6$

Then, substitute $k = 6$ into either of the original equations: $h = 6 + 2 \rightarrow h = 8$

$\Longrightarrow$ Holden: 8; Katya: 6

4. Morristown has 16,000 residents. If its number of residents decreases by 2,000, Morristown will still have more than twice as many residents as Janesville. What is the range of possible numbers of residents Janesville has?

In this scenario, you're looking for a range, so you'll be working with an inequality.

Translate the first sentence as: $m = 16{,}000$

Translate the second sentence as: $m - 2{,}000 > 2j$

The question is: What is j?

Now, substitute $m = 16{,}000$ into the inequality and simplify: $16{,}000 - 2{,}000 > 2j \rightarrow 14{,}000 > 2j$

To isolate j, divide both sides by 2: $7{,}000 > j$

$\Longrightarrow$ Janesville must have less than 7,000 residents

5. Barry's Bus Company has twice as many buses as its nearest competitor, Flora's Fleet Inc. If Flora's Fleet Inc. acquired 3 more buses, it would have one less than Barry's Bus Company. How many buses does Barry's Bus Company have?

Translate the first sentence as: $b = 2f$

Translate the second sentence as: $f + 3 = b - 1$

The question is: What is b?

There's more than one way to solve. Let's isolate f in the second sentence by subtracting 3 from both sides: $f = b - 4$

Now, substitute $f = b - 4$ into the first equation: $b = 2(b - 4)$

Distribute the 2 on the right side: $b = 2b - 8$

Subtract b from both sides, and add 8 to both sides: $0 = b - 8 \rightarrow b = 8 \Longrightarrow$ Barry's Bus Company has 8 buses

The Average

One common type of word problem on the GMAT tests the *average* (or *arithmetic mean*). The basic average formula is:

$$\frac{\textit{sum of terms}}{\textit{number of terms}} = \textit{average}$$

Example: What is the average price of three books that are priced at $13, $15, and $20, respectively?

Add the terms in the set (13 + 15 + 20), and divide by the number of terms (3) to get the average:

$$\frac{13 + 15 + 20}{3} = \frac{48}{3} = \$16$$ The average price is $16.

Often, you'll be asked to find the sum of the terms or the number of terms rather than the average. As long as you know any two pieces of the formula, you can find the third piece.

Common Average: *The average is the most frequently tested statistical concept on the GMAT.*

Example: If you know that three books have an average price of $16, what is the price for the three books combined?

$$\frac{\textit{sum of terms}}{3} = 16$$ Set up the average formula.

$$\textit{sum of terms} = 16 \cdot 3$$ Multiply both sides of the equation by 3.

$$\textit{sum of terms} = \$48$$ Finish out the multiplication.

Here, you don't know the prices of the books individually, but you know that collectively their price is $48.

Drill

1. If three samples weigh 6, 8, and 13 ounces, respectively, what is their average weight?
2. If the average price of six menu items is $8, what is the price of all six items together?
3. Four children are exactly 2, 3, 5, and *x* years old. If the average age of the children is 4.5 years old, what is the value of *x*?
4. If a set of textbooks collectively weighs 52 pounds and the average weight of each book is 4 pounds, how many textbooks are in the set?
5. On May 1^{st}, three plants weighed an average of 5 kilograms. If each plant's weight increased by 2 kilograms over the next two months, what was the average weight of the plants on July 1^{st}?

Drill Solutions

1. If three samples weigh 6, 8, and 13 ounces, respectively, what is their average weight?

Add up the numbers: $6 + 8 + 13 = 27$. Divide by 3 to find the average: $\frac{27}{3} \Longrightarrow 9$ ounces

2. If the average price of six menu items is $8, what is the price of all six items together?

The formula tells you that: $\frac{\textit{sum of prices}}{6} = \$8$

Multiply both sides by 6 to yield *sum of prices* $\Longrightarrow \$48$

3. Four children are exactly 2, 3, 5, and x years old. If the average age of the children is 4.5 years old, what is the value of x?

Set up the equation: $\frac{2 + 3 + 5 + x}{4} = 4.5$

Add like terms in the numerator, and multiply both sides by 4:

$$\frac{10 + x}{4} = 4.5 \rightarrow 10 + x = 18$$

Subtract 10 from both sides $\Longrightarrow x = 8$

4. If a set of textbooks collectively weighs 52 pounds and the average weight of each book is 4 pounds, how many textbooks are in the set?

The formula tells you that: $\frac{52}{\textit{number of textbooks}} = 4$

For simplicity, let's set x = *number of textbooks*. Then, $\frac{52}{x} = 4$

Multiply both sides by x to get $52 = 4x$. Then, divide both sides by 4 $\Longrightarrow x = 13$ textbooks

5. On May 1^{st}, three plants weighed an average of 5 kilograms. If each plant's weight increased by 2 kilograms over the next two months, what was the average weight of the plants on July 1^{st}?

The first sentence tells you that $\frac{\textit{sum of weights}}{3} = 5$. Multiply both sides by 3 to get: *sum of weights* $= 15$.

If each plant's weight increased by 2 kilograms, and there are 3 plants, the sum must go up by $3 \times 2 = 6$ kilograms. Now, the sum is: $15 + 6 = 21$ kilograms.

Then, run the average formula again: $\frac{21}{3} \Longrightarrow 7$ kilograms

Note that you would have gotten the same end result if you'd simply added 2 kilograms to the original average of 5 kilograms. In other words, you could have *assumed* that each plant's weight was exactly 5 kilograms and then added 2 kilograms to each one, giving you a new average of 7 kilograms. This is a helpful strategy on many average problems that we'll cover in depth in Chapter 10: Statistics.

Everyone is Average:
When the individual numbers in a set are irrelevant, you can assume that the numbers are all equal to the average.

GMAT Problems

Following are some sample GMAT Problem Solving questions. These are realistic questions (similar problems have appeared on the test) that make use of what we've covered in this supplement. However, be aware that while the following questions are of the "straightforward math" variety, most GMAT questions have additional features like tricky wording and tough logic. (Don't worry, you'll learn about these features in the book.)

---

1. $|-2|(|-30| - |6|) =$

(A) −72
(B) −48
(C) 48
(D) 60
(E) 72

2. Which of the following represent negative numbers?

I. $-4 - (-7)$

II. $(-4)(-7)$

III. $-7 - (-4)$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

3. $\frac{79.7}{1.92 \times 5.14}$ is approximately

(A) 7
(B) 8
(C) 9
(D) 10
(E) 11

4. If $x(2 - 2.5) = 3$, then $x =$

(A) −600
(B) −210
(C) −6
(D) 6
(E) 600

5. A cable 36.4 feet long is cut into two smaller pieces of cable. If the length of one piece of cable is 3.2 feet longer than the length of the other, what is the length, in feet, of the shorter piece of cable?

(A) 16.6
(B) 17.4
(C) 18.8
(D) 19.8
(E) 20.7

6. A vintage reseller purchased 800 records at a price of $10 each. The reseller sold 500 of the records in January for $24 each and 200 in February for $8 each. The reseller was unable to sell the remaining records. What was the reseller's gross profit on the sale of these records?

(A) $3,000
(B) $4,000
(C) $4,600
(D) $5,100
(E) $5,600

---

1. $|-2|(|-30| - |6|) =$

(A) −72
(B) −48
(C) 48
(D) 60
(E) 72

This question tests your knowledge of PEMDAS and absolute value.

Since the absolute values are nested within the parentheses, resolve them first:

2(30 − 6)	
2(24)	Then, resolve the parentheses.
48	Finally, complete the multiplication.

The answer is C. Confirm your answer before selecting it. (We'll go deep into confirming your answer when I introduce the problem-solving process in Chapter 4: Problem Solving 1. For now, just do a quick double-check every time you get an answer to a GMAT question to make sure you answered the question that was asked and didn't make any careless errors.)

2. Which of the following represent negative numbers?

I. −4 − (− 7)

II. (− 4)(− 7)

III. −7 − (− 4)

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

On a Roman numeral question, you must solve each statement separately—it's like having three problems in one. Work through PEMDAS on each of the Roman numerals.

For Statement I, remember that subtracting a negative is like adding a positive:

$-4 - (-7) \rightarrow -4 + 7 \rightarrow 3$

For Statement II, negative times negative equals positive:

$(-4)(-7) \rightarrow 28$

Like Statement I, Statement III involves subtracting a negative:

$-7 - (-4) \rightarrow -7 + 4 \rightarrow -3$

Only III gives you a negative, so the answer is C. Confirm your answer before selecting it.

3. $\frac{79.7}{1.92 \times 5.14}$ is approximately

(A) 7
(B) 8
(C) 9
(D) 10
(E) 11

While it's possible to calculate the decimals here, doing so will be very tedious. In fact, the question itself contains a clue that you should estimate with the word "approximately."

Note that the given numbers are close to nice round numbers—80 in the numerator and 2 and 5 in the denominator. What can also give you confidence in estimating is the fact that the two numbers in the denominator can be rounded in opposite directions, "balancing" each other out.

You can rewrite the expression as:

$\frac{80}{2 \times 5}$	Replace the decimals with round numbers.
$\frac{80}{10}$	Simplify the denominator.
8	Perform the division.

The answer is B. Confirm your answer before selecting it.

GMAT Solutions (continued)

4. If $x(2 - 2.5) = 3$, then $x =$

(A) −600
(B) −210
(C) −6
(D) 6
(E) 600

This problem has a relatively straightforward setup—you just need to solve for *x*. Let's follow the steps to isolate the variable:

$x(-0.5) = 3$	Combine the numbers within the parentheses.
$x = \frac{3}{-0.5}$	Divide both sides by −0.5.
$x = \frac{30}{-5}$	Move the decimal point over one place in both numbers so that you're dividing by an integer.
$x = -6$	Divide 30 by −5.

The answer is C. Confirm your answer before selecting it.

5. A cable 36.4 feet long is cut into two smaller pieces of cable. If the length of one piece of cable is 3.2 feet longer than the length of the other, what is the length, in feet, of the shorter piece of cable?

(A) 16.6
(B) 17.4
(C) 18.8
(D) 19.8
(E) 20.7

This is a word problem requiring solid translation and equation-creating skills. You can call the shorter piece of cable *x*, which makes the longer piece of cable $x + 3.2$. Then, you know that the two pieces added together total to 36.4:

$x + (x + 3.2) = 36.4$	
$2x + 3.2 = 36.4$	Add the like terms together.
$2x = 33.2$	Subtract 3.2 from both sides.
$x = 16.6$	Divide both sides by 2.

The answer is A. Remember to confirm your answer. Note the trap answer D, which represents the longer piece of cable. It's important to always make sure that you're solving the right question, especially when you're working with word problems.

6. A vintage reseller purchased 800 records at a price of $10 each. The reseller sold 500 of the records in January for $24 each and 200 in February for $8 each. The reseller was unable to sell the remaining records. What was the reseller's gross profit on the sale of these records?

(A) $3,000
(B) $4,000
(C) $4,600
(D) $5,100
(E) $5,600

This problem is a particular type of word problem that requires both proper translation and understanding the concept of profit. The GMAT doesn't test any advanced financial or business concepts, but you do need to understand the basic profit equation: *profit* = *revenue* − *cost*.

The question asks for *gross* profit, but on the GMAT, you aren't expected to make the distinction between gross profit and net profit; knowing the profit formula is enough.

The first sentence tells you how much the records cost:

$800 \times \$10 = \$8{,}000$

The second sentence gives you revenue information, which you can set up for January:

$500 \times \$24$

The calculation could be tedious, but note that using landmark values can help you calculate quickly here. You can instead multiply 1000 by $24 and then divide the result by 2:

$1000 \times \$24 \div 2 \rightarrow \$24{,}000 \div 2 \rightarrow \$12{,}000$

Finally, calculate revenue for February:

$200 \times \$8 = \$1{,}600$

So, the total revenue is:

$\$12{,}000 + \$1{,}600 = \$13{,}600$

Plugging revenue and cost into *profit* = *revenue* − *cost,* you have:

profit = $13,600 − $8,000 = $5,600

The answer is E. Confirm your answer, and you are finished.

We'll go into further depth on profit problems in the book (Chapter 23: Word Problems).

Wrap-Up

Many GMAT questions require math competence at only a basic level—but without that, you'll miss out on points, no matter how solid your other test-taking skills are.

After going through this supplement, how do you feel about the fundamental skills discussed? If you feel comfortable with the skills covered in each section, you're ready to get into the math concepts covered in *The GMAT Mentor: Quant and Data Insights* (available at Amazon, if you don't already have it).

But if you don't, that's okay too. Return to the sections and problems that gave you trouble, and work through them again until they become clear. In fact, this is a great habit to cultivate throughout your GMAT studying: reviewing and redoing concepts and questions until you truly understand them.

Either way, keep in mind that while it's critical to know your math fundamentals, the GMAT isn't really testing your math knowledge. It's really a game of reasoning, with math fundamentals as the "table stakes" required to play the game. *The GMAT Mentor* contains lessons on logical processes and solving methods (as well as further lessons on math fundamentals) that will enable you to use your math skills when and where they'll help you most. Your GMAT journey is just beginning, but we'll get through it together—from foundation all the way to mastery.

GMAT Problems Answer Key

1. C
2. C
3. B
4. C
5. A
6. E